CHAPTER 24 NUCLEAR REACTIONS AND THEIR APPLICATIONS

24.1  
a) Chemical reactions are accompanied by relatively small changes in energy while nuclear reactions are accompanied by relatively large changes in energy. 
b) Increasing temperature increases the rate of a chemical reaction but has no effect on a nuclear reaction. 
c) Both chemical and nuclear reaction rates increase with higher reactant concentrations. 
d) If the reactant is limiting in a chemical reaction, then more reactant produces more product and the yield increases in a chemical reaction. The presence of more radioactive reagent results in more decay product, so a higher reactant concentration increases the yield in a nuclear reaction.

24.2  
a) The percentage of sulfur atoms that are sulfur-32 is 95.02%, the same as the relative abundance of $^{32}\text{S}$. 
b) The atomic mass is larger than the isotopic mass of $^{32}\text{S}$. Sulfur-32 is the lightest isotope, as stated in the problem, so the other 5% of sulfur atoms are heavier than 31.972070 amu. The average mass of all the sulfur atoms will therefore be greater than the mass of a sulfur-32 atom.

24.3  
a) She found that the intensity of emitted radiation is directly proportional to the concentration of the element in the various samples, not to the nature of the compound in which the element occurs. 
b) She found that certain uranium minerals were more radioactive than pure uranium, which implied that they contained traces of one or more as yet unknown, highly radioactive elements. Pitchblende is the principal ore of uranium.

24.4 Radioactive decay that produces a different element requires a change in atomic number ($Z$, number of protons).

\[ \frac{A}{Z}X \rightarrow \frac{A-4}{Z-2}Y + \frac{4}{2}\text{He} \] 2 fewer protons, 2 fewer neutrons

a) Alpha decay produces an atom of a different element, i.e., a daughter with two less protons and two less neutrons.

b) Beta decay produces an atom of a different element, i.e., a daughter with one more proton and one less neutron. A neutron is converted to a proton and $\beta$ particle in this type of decay.

\[ \frac{A}{Z}X \rightarrow \frac{A}{Z+1}Y + \frac{0}{-1}\beta \] 1 more proton, 1 less neutron

c) Gamma decay does not produce an atom of a different element and $Z$ and $N$ remain unchanged.

\[ \frac{A}{Z}X \rightarrow \frac{A}{Z}X + \frac{0}{0}\gamma \] ($\frac{A}{Z}X^*$ = energy rich state), no change in number of protons or neutrons.

d) Positron emission produces an atom of a different element, i.e., a daughter with one less proton and one more neutron. A proton is converted into a neutron and positron in this type of decay.

\[ \frac{A}{Z}X \rightarrow \frac{A}{Z-1}Y + \frac{1}{0}\beta \] 1 less proton, 1 more neutron

e) Electron capture produces an atom of a different element, i.e., a daughter with one less proton and one more neutron. The net result of electron capture is the same as positron emission, but the two processes are different.

\[ \frac{A}{Z}X + \frac{0}{-1}\text{e} \rightarrow \frac{A}{Z+1}Y \] 1 less proton, 1 more neutron

A different element is produced in all cases except (c).

24.5 The key factor that determines the stability of a nuclide is the ratio of the number of neutrons to the number of protons, the $N/Z$ ratio. If the $N/Z$ ratio is either too high or not high enough, the nuclide is unstable and decays.

\[ \frac{1}{2}\text{He} \quad N/Z = 1/2 \]
\[ \frac{2}{2}\text{He} \quad N/Z = 0/2, \text{ thus it is more unstable.} \]
A neutron-rich nuclide decays to convert neutrons to protons while a neutron-poor nuclide decays to convert protons to neutrons. The conversion of neutrons to protons occurs by beta decay:

\[ {}^1_0n \rightarrow {}^1_1p + {}^0_{-1}\beta \]

The conversion of protons to neutrons occurs by either positron decay:

\[ {}^1_1p \rightarrow {}^0_{-1}n + {}^0_{+1}\beta \]

or electron capture:

\[ {}^1_1p + {}^0_{-1}e \rightarrow {}^0_{-1}n \]

Neutron-rich nuclides, with a high \( N/Z \), undergo \( \beta^- \) decay. Neutron-poor nuclides, with a low \( N/Z \), undergo positron decay or electron capture.

Both positron emission and electron capture increase the number of neutrons and decrease the number of protons. The products of both processes are the same. Positron emission is more common than electron capture among lighter nuclei; electron capture becomes increasingly common as nuclear charge increases. For \( Z < 20 \), \( \beta^+ \) emission is more common; for \( Z > 80 \), \( e^- \) capture is more common.

In a balanced nuclear equation, the total of mass numbers and the total of charges on the left side and the right side must be equal.

a) \( ^{234}_{92}U \rightarrow ^4_2He + ^{230}_{90}Th \)  
Mass: 234 = 4 + 230;  
Charge: 92 = 2 + 90

b) \( ^{232}_{93}Np + ^0_{-1}e \rightarrow ^{232}_{92}U \)  
Mass: 232 + 0 = 232;  
Charge: 93 + (–1) = 92

c) \( ^{12}_7N \rightarrow ^0_{+1}\beta + ^{12}_6C \)  
Mass: 12 = 0 + 12;  
Charge: 7 = 1 + 6

a) \( ^{26}_{11}Na \rightarrow ^0_{+1}\beta + ^{26}_{12}Mg \)

b) \( ^{223}_{87}Fr \rightarrow ^0_{+1}\beta + ^{223}_{88}Ra \)

c) \( ^{212}_{83}Bi \rightarrow ^4_2\alpha + ^{208}_{81}Tl \)

The process converts a neutron to a proton, so the mass number is the same, but the atomic number increases by one.

\[ ^{27}_{12}Mg \rightarrow ^0_{+1}\beta + ^{27}_{13}Al \]

b) Positron emission decreases atomic number by one, but not mass number.

\[ ^{23}_{12}Mg \rightarrow ^0_{+1}\beta + ^{23}_{11}Na \]

c) The electron captured by the nucleus combines with a proton to form a neutron, so mass number is constant, but atomic number decreases by one.

\[ ^{103}_{46}Pd + ^0_{-1}e \rightarrow ^{103}_{45}Rh \]

a) \( ^{32}_{14}Si \rightarrow ^0_{-1}\beta + ^{32}_{15}P \)

b) \( ^{218}_{84}Po \rightarrow ^4_2\alpha + ^{214}_{82}Pb \)

c) \( ^{110}_{49}\text{In} + ^0_{-1}e \rightarrow ^{110}_{48}\text{Cd} \)

a) In other words, an unknown nuclide decays to give Ti–48 and a positron.

\[ ^{48}_{21}V \rightarrow ^{48}_{22}\text{Ti} + ^0_{+1}\beta \]

b) In other words, an unknown nuclide captures an electron to form Ag–107.

\[ ^{107}_{46}\text{Cd} + ^0_{-1}e \rightarrow ^{107}_{47}\text{Ag} \]

c) In other words, an unknown nuclide decays to give Po–206 and an alpha particle.

\[ ^{210}_{86}\text{Rn} \rightarrow ^{206}_{84}\text{Po} + ^4_2\text{He} \]
24.13 a) $^{241}_{94}\text{Pu} \rightarrow ^{241}_{95}\text{Am} + 0\beta$
   b) $^{228}_{88}\text{Ra} \rightarrow ^{228}_{89}\text{Ac} + 0\beta$
   c) $^{207}_{85}\text{At} \rightarrow ^{203}_{83}\text{Bi} + 4\alpha$

24.14 a) $^{186}_{78}\text{Pt} + 0\text{e} \rightarrow ^{186}_{77}\text{Ir}$
   b) $^{225}_{89}\text{Ac} \rightarrow ^{221}_{87}\text{Fr} + 4\alpha$
   c) $^{129}_{52}\text{Te} \rightarrow ^{129}_{53}\text{I} + 0\beta$

24.15 a) $^{52}_{26}\text{Fe} \rightarrow ^{52}_{25}\text{Mn} + 0\beta$
   b) $^{219}_{86}\text{Rn} \rightarrow ^{215}_{84}\text{Po} + 4\alpha$
   c) $^{37}_{18}\text{Rb} + 0\text{e} \rightarrow ^{36}_{18}\text{Kr}$

24.16 Look at the $N/Z$ ratio, the ratio of the number of neutrons to the number of protons. If the $N/Z$ ratio falls in the band of stability, the nuclide is predicted to be stable. For stable nuclides of elements with atomic number greater than 20, the ratio of number of neutrons to number of protons ($N/Z$) is greater than one. In addition, the ratio increases gradually as atomic number increases. Also check for exceptionally stable numbers of neutrons and/or protons -- the “magic” number of 2, 8, 20, 28, 50, 82 and (N = 126). Also, even numbers of protons and/or neutrons are related to stability whereas odd numbers are related to instability.

(a) $^{20}_{8}\text{O}$ appears stable because its $Z$ (8) value is a magic number, but its $N/Z$ ratio $(20 - 8)/8 = 1.50$ is too high and this nuclide is above the band of stability; $^{20}_{8}\text{O}$ is unstable.

(b) $^{59}_{27}\text{Co}$ might look unstable because its $Z$ value is an odd number, but its $N/Z$ ratio $(59 - 27)/27 = 1.19$ is in the band of stability, so $^{59}_{27}\text{Co}$ appears stable.

(c) $^{9}_{3}\text{Li}$ appears unstable because its $N/Z$ ratio $(9 - 3)/3 = 2.00$ is too high and is above the band of stability.

24.17 a) $^{146}_{60}\text{Nd}$  $N/Z = 86/60 = 1.4$ Stable, $N/Z$ ok
   b) $^{114}_{48}\text{Cd}$  $N/Z = 66/48 = 1.4$ Stable, $N/Z$ ok
   c) $^{88}_{42}\text{Mo}$  $N/Z = 46/42 = 1.1$ Unstable, $N/Z$ too small for this region of the band

24.18 For stable nuclides of elements with atomic number greater than 20, the ratio of number of neutrons to number of protons ($N/Z$) is greater than one. In addition, the ratio increases gradually as atomic number increases.
   a) For the element iodine $Z = 53$. For iodine-127, $N = 127 - 53 = 74$. The $N/Z$ ratio for $^{127}\text{I}$ is $74/53 = 1.4$. Of the examples of stable nuclides given in the book $^{107}\text{Ag}$ has the closest atomic number to iodine. The $N/Z$ ratio for $^{107}\text{Ag}$ is 1.3. Thus, it is likely that iodine with 6 additional protons is stable with an $N/Z$ ratio of 1.4.
   b) Tin is element number 50 ($Z = 50$). From part (a), the stable nuclides of tin would have $N/Z$ ratios approximately 1.3 to 1.4. The $N/Z$ ratio for $^{106}\text{Sn}$ is $(106 - 50)/50 = 1.1$. The nuclide $^{106}\text{Sn}$ is unstable with an $N/Z$ ratio that is too low.
   c) For $^{68}_{33}\text{As}$, $Z = 33$ and $N = 68 - 33 = 35$ and $N/Z = 1.1$. The ratio is within the range of stability, but the nuclide is most likely unstable because there is an odd number of both protons and neutrons.

24.19 a) $^{48}_{10}\text{K}$  $N/Z = 29/19 = 1.5$ Unstable, $N/Z$ too large for this region of the band
   b) $^{79}_{35}\text{Br}$  $N/Z = 44/35 = 1.3$ Stable, $N/Z$ okay
   c) $^{33}_{18}\text{Ar}$  $N/Z = 14/18 = 0.78$ Unstable, $N/Z$ too small
24.20  a) $^{238}_92\text{U}$: Nuclides with $Z > 83$ decay through $\alpha$ decay.

b) The $N/Z$ ratio for $^{48}_24\text{Cr}$ is $(48 - 24)/24 = 1.00$. This number is below the band of stability because $N$ is too low and $Z$ is too high. To become more stable, the nucleus decays by converting a proton to a neutron, which is positron decay. Alternatively, a nucleus can capture an electron and convert a proton into a neutron through electron capture.

c) The $N/Z$ ratio for $^{50}_25\text{Mn}$ is $(50 - 25)/25 = 1.00$. This number is also below the band of stability, so the nuclide undergoes positron decay or electron capture.

24.21  a) $^{111}_47\text{Ag}$ beta decay $N/Z = 1.4$ which is too high

b) $^{41}_17\text{Cl}$ beta decay $N/Z = 1.4$ which is too high

c) $^{110}_44\text{Ru}$ beta decay $N/Z = 1.5$ which is too high

24.22  a) For carbon-15, $N/Z = 9/6 = 1.5$, so the nuclide is neutron-rich. To decrease the number of neutrons and increase the number of protons, carbon-15 decays by beta decay.

b) The $N/Z$ ratio for $^{120}_54\text{Xe}$ is $66/54 = 1.2$. Around atomic number 50, the ratio for stable nuclides is larger than 1.2, so $^{120}_54\text{Xe}$ is proton-rich. To decrease the number of protons and increase the number of neutrons, the xenon-120 nucleus either undergoes positron emission or electron capture.

c) Thorium-224 has an $N/Z$ ratio of $134/90 = 1.5$. All nuclides of elements above atomic number 83 are unstable and decay to decrease the number of both protons and neutrons. Alpha decay by thorium-224 is the most likely mode of decay.

24.23  a) $^{106}_49\text{In}$ positron decay or electron capture $N/Z = 1.2$

b) $^{141}_63\text{Eu}$ positron decay or electron capture $N/Z = 1.2$

c) $^{241}_95\text{Am}$ alpha decay $N/Z = 1.5$

24.24 Stability results from a favorable $N/Z$ ratio, even numbers of $N$ and/or $Z$, and the occurrence of magic numbers. The $N/Z$ ratio of $^{52}_24\text{Cr}$ is $(52 - 24)/24 = 1.17$, which is within the band of stability. The fact that $Z$ is even does not account for the variation in stability because all isotopes of chromium have the same $Z$. However, $^{52}_24\text{Cr}$ has 28 neutrons, so $N$ is both an even number and a magic number for this isotope only.

24.25 $^{40}_20\text{Ca}$ $N/Z = 20/20 = 1.0$

It lies in the band of stability, and $N$ and $Z$ are both even and magic.

24.26 $^{237}_93\text{Np} \rightarrow 4\alpha + 233_91\text{Pa}$

$^{233}_91\text{Pa} \rightarrow 0\beta + 233_92\text{U}$

$^{233}_92\text{U} \rightarrow 4\alpha + 229_90\text{Th}$

$^{229}_90\text{Th} \rightarrow 4\alpha + 225_88\text{Ra}$

24.27 Alpha emission produces helium ions which readily pick up electrons to form stable helium atoms.

24.28 The equation for the nuclear reaction is $^{235}_92\text{U} \rightarrow 207_82\text{Pb} + 0\beta + 4\alpha$ to determine the coefficients, notice that the beta particles will not impact the mass number. Subtracting the mass number for lead from the mass number for uranium will give the total mass number for the alpha particles released, $235 - 207 = 28$. Each alpha particle is a helium nucleus with mass number 4. The number of helium atoms is determined by dividing the total mass number change by 4, $28/4 = 7$ helium atoms or 7 alpha particles.
The equation is now
\[ ^{235}_{92}U \rightarrow ^{207}_{82}Pb + \_0^1\beta + 7\_2^4He \]
To find the number of beta particles released, examine the difference in number of protons (atomic number) between the reactant and products. Uranium, the reactant, has 92 protons. The atomic number in the products, lead atom and 7 helium nuclei, total 96. To balance the atomic numbers, four electrons (beta particles) must be emitted to give the total atomic number for the products as 96 – 4 = 92, the same as the reactant.
In summary, 7 alpha particles and 4 beta particles are emitted in the decay of uranium-235 to lead-207.
\[ ^{235}_{92}U \rightarrow ^{207}_{82}Pb + 4\ _0^1\beta + 7\_2^4He \]

24.29 a) In a scintillation counter, radioactive emissions are detected by their ability to excite atoms and cause them to emit light.
b) In a Geiger-Müller counter, radioactive emissions produce ionization of a gas that conducts a current to a recording device.

24.30 Since the decay rate depends only on the number of radioactive nuclei, radioactive decay is a first-order process.

24.31 No, it is not valid to conclude that \( t_{1/2} \) equals 1 minute because the number of nuclei is so small (6 nuclei). Decay rate is an average rate and is only meaningful when the sample is macroscopic and contains a large number of nuclei, as in the second case. Because the second sample contains \( 6 \times 10^{12} \) nuclei, the conclusion that \( t_{1/2} = 1 \) minute is valid.

24.32 High-energy neutrons in cosmic rays enter the upper atmosphere and keep the amount of \(^{14}\)C nearly constant through bombardment of ordinary \(^{14}\)N atoms. This \(^{14}\)C is absorbed by living organisms, so its proportion stays relatively constant there also.
\[ ^{14}_{7}N + ^{0}_{1}n \rightarrow ^{14}_{6}C + ^{1}_{1}H \]

24.33 Specific activity of a radioactive sample is its decay rate per gram. Calculate the specific activity from the number of particles emitted per second (disintegrations per second = dps) and the mass of the sample.
Specific activity = decay rate per gram.
\[ 1 \text{ Ci} = 3.70 \times 10^{10} \text{ dps} \]
Specific Activity = \[ \left( \frac{1.56 \times 10^6 \text{ dps}}{1.65 \text{ mg}} \right) \left( \frac{1 \text{ mg}}{10^{-3} \text{ g}} \right) \left( \frac{1 \text{ Ci}}{3.70 \times 10^{10} \text{ dps}} \right) = 2.5528 \times 10^{-2} = 2.56 \times 10^{-2} \text{ Ci/g} \]

24.34 Specific Activity = \[ \left( \frac{4.13 \times 10^8 \text{ d}}{2.6 \text{ g}} \right) \left( \frac{1 \text{ h}}{3600 \text{ s}} \right) \left( \frac{1 \text{ Ci}}{3.70 \times 10^{10} \text{ dps}} \right) = 1.1925 \times 10^{-6} = 1.2 \times 10^{-6} \text{ Ci/g} \]

24.35 A becquerel is a disintegration per second (d/s). A disintegration can be any of the radioactive decay particles mentioned in the text (\( \alpha \), \( \beta \), positron, etc.). Therefore, the emission of one \( \alpha \) particle equals one disintegration.
Specific Activity = \[ \left( \frac{7.4 \times 10^4 \text{ d}}{8.58 \mu g} \right) \left( \frac{1 \text{ min}}{60 \text{ s}} \right) \left( \frac{1 \text{ bq}}{1 \text{ dps}} \right) = 1.43745 \times 10^8 = 1.4 \times 10^8 \text{ Bq/g} \]
Specific Activity = \left( \frac{3.77 \times 10^7 \text{ d}}{1 \text{ min}} \right) \left( \frac{1 \text{ min}}{60 \text{ s}} \right) \left( \frac{1 \text{ bq}}{1 \text{ dps}} \right) = 587.2274 = 587 \text{ Bq/g}

Decay constant is the rate constant for the first order reaction:
Decay rate = \frac{-\Delta N}{\Delta t} = kN
\quad \Rightarrow k = 1 \times 10^{-12} \text{ d}^{-1}

Decay rate = \frac{-2.8 \times 10^{-12} \text{ atom}}{1 \text{ yr}} = k (1 \text{ atom})
\quad \Rightarrow k = 2.8 \times 10^{-12} \text{ yr}^{-1}

The rate constant, $k$, relates the number of radioactive nuclei to their decay rate through the equation $A = kN$. The number of radioactive nuclei is calculated by converting moles to atoms using Avogadro’s number. The decay rate is $1.39 \times 10^5 \text{ d/yr}$ or more simply, $1.39 \times 10^5 \text{ yr}^{-1}$ (the disintegrations are assumed).

Decay rate = \frac{-1.39 \times 10^5 \text{ atom}}{1 \text{ yr}} = k (6.022 \times 10^{23} \text{ atom})
\quad \Rightarrow k = 2.30820 \times 10^{-7} \text{ yr}^{-1}

Radioactive decay is a first-order process, so the integrated rate law is $\ln[N]_t = \ln[N]_0 - kt$
To calculate the fraction of bismuth-212 remaining after $3.75 \times 10^3 \text{ h}$, first find the value of $k$ from the half-life, then calculate the fraction remaining with $N_0$ set to 1 (exactly).

$t_{1/2} = 1.01 \text{ yr}$
\quad t = 3.75 \times 10^3 \text{ h}
\quad \Rightarrow k = \frac{(\ln 2)}{(t_{1/2})} = \frac{(\ln 2)}{(1.01 \text{ yr})} = 0.686284 \text{ yr}^{-1}$ (unrounded)
\quad \ln[N]_t = \ln[N]_0 - kt
\quad \ln[N]_t = \ln[2.00 \text{ mg}] - (0.686284 \text{ yr}^{-1})(3.75 \times 10^3 \text{ h})\left(\frac{1 \text{ d}}{24 \text{ h}}\right)\left(\frac{1 \text{ y}}{365 \text{ d}}\right)
\quad \ln[N]_t = 0.399361
\quad N_t = e^{0.399361}
\quad N_t = 1.49087 = 1.49 \text{ mg}

$t_{1/2} = 1.60 \times 10^3 \text{ yr}$
\quad t = ? \text{ h}
\quad k = \frac{(\ln 2)}{(t_{1/2})} = \frac{(\ln 2)}{(1.60 \times 10^3 \text{ yr})} = 0.000433216 \text{ yr}^{-1}$ (unrounded)
\quad \ln \frac{2.50 \text{ g}}{0.185 \text{ g}} = \left[\frac{(0.000433216 \text{ yr}^{-1}) (1 \text{ yr} / 365 \text{ day}) (1 \text{ day} / 24 \text{ h})}{t}\right]
\quad 2.603690 = 4.945399 \times 10^{-8} \text{ h}^{-1} \cdot t$ (unrounded)
\quad t = \frac{(2.603690)}{(4.945399 \times 10^{-8} \text{ h}^{-1})} = 5.2648734 \times 10^7 = 5.26 \times 10^7 \text{ h}
24.43 Lead–206 is a stable daughter of $^{238}$U. Since all of the $^{206}$Pb came from $^{238}$U, the starting amount of $^{238}$U was $(270 \, \mu\text{mol} + 110 \, \mu\text{mol}) = 380 \, \mu\text{mol} = N_0$. The amount of $^{238}$U at time $t$ (current) is $270 \, \mu\text{mol} = N_t$. Find $k$ from the first-order rate expression for half-life, and then substitute the values into the integrated rate law and solve for $t$.

$$t_{1/2} = \frac{\ln 2}{k} \quad k = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{(4.5 \times 10^9 \text{ yr})} = 1.540327 \times 10^{-10} \text{ yr}^{-1} \quad \text{(unrounded)}$$

$$\ln \frac{N_0}{N_t} = kt$$

$$\ln \frac{380 \, \mu\text{mol}}{270 \, \mu\text{mol}} = (1.540327 \times 10^{-10} \text{ yr}^{-1})t$$

$$0.341749293 = (1.540327 \times 10^{-10} \text{ yr}^{-1})t$$

$$t = \frac{0.341749293}{(1.540327 \times 10^{-10} \text{ yr}^{-1})} = 2.21868 \times 10^9 = 2.2 \times 10^9 \text{ yr}$$

24.44 The ratio $(0.735)$ equals $N_t / N_0$.

$$\frac{N_0}{N_t} = 1.360544218 \quad \text{(unrounded)}$$

$$t_{1/2} = \frac{\ln 2}{k} \quad k = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{(5730 \text{ yr})} = 1.2096809 \times 10^{-4} \text{ yr}^{-1} \quad \text{(unrounded)}$$

$$\ln \frac{N_0}{N_t} = \frac{\ln 1.360544218}{(1.2096809 \times 10^{-4} \text{ yr}^{-1})t}$$

$$0.30788478 = (1.2096809 \times 10^{-4} \text{ yr}^{-1})t$$

$$t = \frac{0.30788478}{(1.2096809 \times 10^{-4} \text{ yr}^{-1})} = 2.54517 \times 10^3 = 2.54 \times 10^3 \text{ yr}$$

24.45 Use the conversion factor, $1 \text{ Ci} = 3.70 \times 10^{10}$ disintegrations per second (dps).

$$\text{Activity} = \left(6 \times 10^{-11} \text{ mCi} \right) \left(10^{-3} \text{ Ci} \right) \left(3.70 \times 10^{10} \text{ dps} \right) \left(\frac{60 \text{ s}}{1 \text{ min}}\right) \left(\frac{1000 \text{ mL}}{1 \text{ qt}}\right) \left(\frac{1 \text{ qt}}{4 \text{ cups}}\right) \left(\frac{1 \text{ cup}}{8 \text{ oz}}\right)$$

$$= 31.50426 = 30 \text{ dpm}$$

24.46 Plutonium-239 ($t_{1/2} = 2.41 \times 10^4 \text{ yr}$)

Time $= 7 \, t_{1/2} = 7 \, (2.41 \times 10^4 \text{ yr}) = 1.6870 \times 10^5 = 1.69 \times 10^5 \text{ yr}$

24.47 Both $N_t$ and $N_0$ are given: the number of nuclei present currently, $N_t$, is found from the moles of $^{232}$Th. Each fission track represents one nucleus that disintegrated, so the number of nuclei disintegrated is added to the number of nuclei currently present to determine the initial number of nuclei, $N_0$. The rate constant, $k$, is calculated from the half-life. All values are substituted into the first-order decay equation to find $t$.

$$t_{1/2} = \frac{\ln 2}{k} \quad k = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{(1.4 \times 10^{10} \text{ yr})} = 4.9510512 \times 10^{-11} \text{ yr}^{-1} \quad \text{(unrounded)}$$

$$N_t = (3.1 \times 10^{-15} \text{ mol Th}) (6.022 \times 10^{23} \text{ atoms Th / mol Th}) = 1.86682 \times 10^9 \text{ atoms Th} \quad \text{(unrounded)}$$

$$\ln \frac{N_0}{N_t} = kt$$

$$\ln \left(\frac{1.86682 \times 10^9 + 9.5 \times 10^3}{1.86682 \times 10^9}\right) = (4.9510512 \times 10^{-11} \text{ yr}^{-1})t$$

$$5.08874 \times 10^{-5} = (4.9510512 \times 10^{-11} \text{ yr}^{-1})t$$

$$t = \frac{5.08874 \times 10^{-5}}{(4.9510512 \times 10^{-11} \text{ yr}^{-1})} = 1.027809 \times 10^6 = 1.0 \times 10^6 \text{ yr}$$
24.48 The mole relationship between $^{40}$K and $^{40}$Ar is 1:1. Thus, 1.14 mmol $^{40}$Ar = 1.14 mmol $^{40}$K decayed.

$$t_{1/2} = \frac{\ln(2)}{k}$$

$$k = \frac{\ln(2)}{t_{1/2}} = \frac{\ln(2)}{(1.25 \times 10^9 \text{ yr})} = 5.5451774 \times 10^{-10} \text{ yr}^{-1} \text{ (unrounded)}$$

$$\ln \left( \frac{N_0}{N_t} \right) = kt$$

$$\ln \left( \frac{(1.38 + 1.14) \text{ mmol}}{1.38 \text{ mmol}} \right) = (5.5451774 \times 10^{-10} \text{ yr}^{-1})t$$

$$0.6021754 = (5.5451774 \times 10^{-10} \text{ yr}^{-1})t$$

$$t = \frac{0.6021754}{(5.5451774 \times 10^{-10} \text{ yr}^{-1})} = 1.08594 \times 10^9 = 1.09 \times 10^9 \text{ yr}$$

24.49 $^{27}$Al + $^4$He → $^{30}$P + $^1_0$n

They experimentally confirmed the existence of neutrons, and were the first to produce an artificial radioisotope.

24.50 Both gamma radiation and neutron beams have no charge, so neither is deflected by electric or magnetic fields. Neutron beams differ from gamma radiation in that a neutron has mass approximately equal to that of a proton. Researchers observed that a neutron beam could induce the emission of protons from a substance. Gamma rays do not cause such emissions.

24.51 A proton, for example, exits the first tube just when it becomes positive and the next tube becomes negative. Pushed by the first tube and pulled by the second, the proton accelerates across the gap between them.

24.52 Protons are repelled from the target nuclei due to the interaction of like (positive) charges. Higher energy is required to overcome the repulsion.

24.53 a) An alpha particle is a reactant with $^{10}$B and a neutron is one product. The mass number for the reactants is $10 + 4 = 14$. So, the missing product must have a mass number of $14 - 1 = 13$. The total atomic number for the reactants is $5 + 2 = 7$, so the atomic number for the missing product is 7.

$$^{10}_5\text{B} + ^4_2\text{He} \rightarrow ^1_0\text{n} + ^{13}_7\text{N}$$

b) For the reactants, the mass number is $28 + 2 = 30$ and the atomic number is $14 + 1 = 15$. The given product has mass number 29 and atomic number 15, so the missing product particle has mass number 1 and atomic number 0. The particle is thus a neutron.

$$^{28}_{14}\text{Si} + ^2_1\text{H} \rightarrow ^1_0\text{n} + ^{29}_{15}\text{P}$$

c) The products are 2 neutrons and $^{244}$Cf with a total mass number of $2 + 244 = 246$, and an atomic number of 98. The given reactant particle is an alpha particle with mass number 4 and atomic number 2. The missing reactant must have mass number of $246 - 4 = 242$ and atomic number $98 - 2 = 96$. Element 96 is Cm.

$$^{242}_{96}\text{Cm} + ^4_2\text{He} \rightarrow 2^1_0\text{n} + ^{244}_{94}\text{Cf}$$

24.54 a) $^{31}_{15}\text{P} + \gamma \rightarrow ^1_1\text{H} + ^1_0\text{n} + ^{29}_{14}\text{Si}$

$$^{31}_{15}\text{P} (\gamma, \text{pn})^{29}_{14}\text{Si}$$

b) $^{252}_{98}\text{Cf} + ^{10}_5\text{B} \rightarrow ^5_4\text{n} + ^{257}_{103}\text{Lr}$

$$^{252}_{98}\text{Cf} (\text{B}, 5 \text{n})^{257}_{103}\text{Lr}$$

c) $^{238}_{92}\text{U} + ^4_2\text{He} \rightarrow ^3_0\text{n} + ^{239}_{94}\text{Pu}$

$$^{238}_{92}\text{U} (\alpha, 3 \text{n})^{239}_{94}\text{Pu}$$

24.55 a) $^{249}_{98}\text{Cf} + ^{12}_6\text{C} \rightarrow ^{257}_{104}\text{Rf} + 4^1_0\text{n}$

$$^{249}_{98}\text{Cf} (\text{C}, 4 \text{n})^{257}_{104}\text{Rf}$$

b) $^{240}_{98}\text{Cf} (^{12}_6\text{C}, 4 \text{n})^{257}_{104}\text{Rf}$
Gamma radiation has no mass or charge while alpha particles are massive and highly charged. These differences account for the different effect on matter that these two types of radiation have. Alpha particles interact with matter more strongly than gamma particles due to their mass and charge. Therefore alpha particles penetrate matter very little. Gamma rays interact very little with matter due to the lack of mass and charge. Therefore gamma rays penetrate matter more extensively.

In the process of ionization, collision of matter with radiation dislodges an electron. The free electron and the positive ion that result are referred to as an ion-pair.

Ionizing radiation is more dangerous to children because their rapidly dividing cells are more susceptible to radiation than an adult’s slowly dividing cells.

The hydroxyl free radical forms more free radicals which go on to attack and change surrounding biomolecules, whose bonding and structure are delicately connected with their function. These changes are irreversible, as opposed to the reversible changes produced by OH.

a) The rad is the amount of radiation energy absorbed in J per body mass in kg.

\[
[3.3 \times 10^{-7} \text{ J} \left( \frac{2.205 \text{ lb}}{1 \text{ kg}} \right) \left( \frac{1 \text{ rad}}{1 \times 10^{-2} \text{ J/kg}} \right)] = 5.39 \times 10^{-7} = 5.4 \times 10^{-7} \text{ rad}
\]

b) Conversion factor is 1 rad = 0.01 Gy

\[
(5.39 \times 10^{-7} \text{ rad}) \left( \frac{0.01 \text{ Gy}}{1 \text{ rad}} \right) = 5.39 \times 10^{-9} = 5.4 \times 10^{-9} \text{ Gy}
\]

a) Dose = \( (8.92 \times 10^{-4} \text{ Gy}) \left( \frac{1 \text{ rad}}{0.01 \text{ Gy}} \right) = 0.0892 \text{ rad} \)

b) Energy = \( (0.0892 \text{ rad}) \left( \frac{0.01 \text{ J/kg}}{1 \text{ rad}} \right) (3.6 \text{ kg}) = 3.2112 \times 10^{-3} = 3.2 \times 10^{-3} \text{ J} \)

a) Convert the given information to units of J/kg.

\[
Dose = \left( \frac{6.0 \times 10^{5} \beta}{70. \text{ kg}} \right) \left( \frac{8.74 \times 10^{-14} \text{ J/} \beta}{1 \text{ rad}} \right) \left( \frac{0.01 \text{ Gy}}{1 \text{ rad}} \right) = 7.4914 \times 10^{-10} = 7.5 \times 10^{-10} \text{ Gy}
\]

b) Convert grays to rads and multiply rads by RBE to find rems. Convert rems to mrems.

\[
\text{rem} = \text{rads} \times \text{RBE} = \left( 7.4914 \times 10^{-10} \text{ Gy} \right) \left( \frac{1 \text{ rad}}{0.01 \text{ Gy}} \right) \left( \frac{1 \text{ mrem}}{1 \times 10^{-3} \text{ rem}} \right) = 7.4914 \times 10^{-5} = 7.5 \times 10^{-5} \text{ mrem}
\]

c) \( 1 \text{ mrem} = 0.01 \text{ Sv} \)

\[
\text{Sv} = (7.5 \times 10^{-5} \text{ mrem}) \left( \frac{10^{-3} \text{ rem}}{1 \text{ mrem}} \right) \left( \frac{0.01 \text{ Sv}}{1 \text{ rem}} \right) = 7.4914 \times 10^{-10} = 7.5 \times 10^{-10} \text{ Sv}
\]

a) Dose = \( (1.77 \times 10^{10} \beta) \left[ \frac{2.20 \times 10^{-12} \text{ J/} \beta}{265 \text{ g}} \right] \left( \frac{1 \text{ rad}}{0.01 \text{ J/kg}} \right) = 1.46943 = 1.47 \text{ rad} \)

b) Dose = \( (1.46943 \text{ rad}) \left( \frac{0.01 \text{ Gy}}{1 \text{ rad}} \right) = 1.46943 \times 10^{-2} = 1.47 \times 10^{-2} \text{ Gy} \)

c) Dose = \( (1.46943 \text{ rad}) \left( \frac{0.75 \text{ rem}}{1 \text{ rad}} \right) \left( \frac{0.01 \text{ Sv}}{1 \text{ rem}} \right) = 1.10207 \times 10^{-2} = 1.10 \times 10^{-2} \text{ Sv} \)

Dose = \( (1.8796973 \times 10^{-8} \text{ rad}) \left( \frac{0.01 \text{ Gy}}{1 \text{ rad}} \right) = 1.8796973 \times 10^{-10} = 1.9 \times 10^{-10} \text{ Gy} \)
24.65 Use the time and disintegrations per second (Bq) to find the number of $^{60}$Co atoms that disintegrate, which equals the number of $\beta$ particles emitted. The dose in rads is calculated as energy absorbed per body mass.

$$Dose = \left( \frac{475 \text{ Bq}}{1.858 \text{ g}} \right) \left( \frac{10^3 \text{ g}}{1 \text{ kg}} \right) \left( \frac{1 \text{ dps}}{1 \text{ Bq}} \right) \left( \frac{5.05 \times 10^{-14} \text{ J}}{1 \text{ disint.}} \right) \left( \frac{24.0 \text{ min}}{1 \text{ Bq}} \right) \left( \frac{60 \text{ s}}{1 \text{ min}} \right) \left( \frac{1 \text{ rad}}{0.01 \text{ J/kg}} \right) = 1.8591 \times 10^{-3} = 1.86 \times 10^{-3} \text{ rad}$$

24.66 A healthy thyroid gland incorporates dietary I$^{-}$ into I-containing hormones at a known rate. To assess thyroid function, the patient drinks a solution containing a trace amount of Na$^{131}$I, and a scanning monitor follows the uptake of $^{131}$I into the thyroid. Technetium-99 is often used for imaging the heart, lungs, and liver.

24.67 NAA does not destroy the sample while chemical analysis does. Neutrons bombard a non-radioactive sample, “activating” or energizing individual atoms within the sample to create radioisotopes. The radioisotopes decay back to their original state (thus, the sample is not destroyed) by emitting radiation that is different for each isotope.

24.68 In positron-emission tomography (PET), the isotope emits positrons, each of which annihilates a nearby electron. In the process, two $\gamma$ photons are emitted simultaneously, 180° apart from each other. Detectors locate the sites and the image is analyzed by computer.

24.69 The concentration of $^{59}$Fe in the steel sample and the volume of oil would be needed.

24.70 The oxygen in formaldehyde comes from methanol because the oxygen isotope in the methanol reactant appears in the formaldehyde product. The oxygen isotope in the chromic acid reactant appears in the water product, not the formaldehyde product. The isotope traces the oxygen in methanol to the oxygen in formaldehyde.

24.71 The mass change in a chemical reaction was considered too minute to be significant and too small to measure with even the most sophisticated equipment.

24.72 When a nucleus forms from its nucleons, there is a decrease in mass called the mass defect. This decrease in mass is due to mass being converted to energy to hold the nucleus together. This energy is called the binding energy.

24.73 Energy is released when a nuclide forms from nucleons. The nuclear binding energy is the amount of energy holding the nucleus together. Energy is absorbed to break the nucleus into nucleons and is released when nucleons “come together.”

24.74 The binding energy per nucleon is the average amount of energy per each component (proton and neutron) part of the nuclide. The binding energies per nucleon are helpful in comparing the stabilities of different combinations, to provide information on the potential processes a nuclide can undergo to become more stable. The binding energy per nucleon varies considerably. The greater the binding energy per nucleon, the more strongly the nucleons are held together and the more stable the nuclide.

24.75 Apply the appropriate conversions from the chapter or the inside back cover.

a) Energy = \( (0.01861 \text{ MeV}) \left( \frac{10^6 \text{ eV}}{1 \text{ MeV}} \right) = 1.861 \times 10^4 \text{ eV} \)

b) Energy = \( (0.01861 \text{ MeV}) \left( \frac{10^6 \text{ eV}}{1 \text{ MeV}} \right) \left( \frac{1.602 \times 10^{-19} \text{ J}}{1 \text{ eV}} \right) = 2.981322 \times 10^{-15} = 2.981 \times 10^{-15} \text{ J} \)

24.76 a) \( (1.57 \times 10^{-15} \text{ kJ}) \left( \frac{10^3 \text{ J}}{1 \text{ kJ}} \right) \left( \frac{1 \text{ eV}}{1.602 \times 10^{-19} \text{ J}} \right) = 9.8002 \times 10^6 = 9.80 \times 10^6 \text{ eV} \)

  b) \( (1.57 \times 10^{-15} \text{ kJ}) \left( \frac{10^3 \text{ J}}{1 \text{ kJ}} \right) \left( \frac{1 \text{ eV}}{1.602 \times 10^{-19} \text{ J}} \right) \left( \frac{1 \text{ MeV}}{10^6 \text{ eV}} \right) = 9.8002 = 9.80 \text{ MeV} \)

24-10
24.77 Convert moles of $^{239}$Pu to atoms of $^{239}$Pu using Avogadro’s number. Multiply the number of atoms by the energy per atom (nucleus) and convert the MeV to J using the conversion $1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$.

$\text{Energy} = \left(1.5 \text{ mol}^{239}\text{Pu}\right) \left(6.022 \times 10^{23} \text{ atoms mol}^{-1}\right) \left(5.243 \text{ MeV} \text{ atom}^{-1}\right) \left(10^6 \text{ eV} \text{ MeV}^{-1}\right) \left(1.602 \times 10^{-19} \text{ J} \text{ eV}^{-1}\right)$

$= 7.587075 \times 10^{11} = 7.6 \times 10^{11} \text{ J}$

24.78 $\text{Energy} = \left(8.11 \times 10^5 \text{ kJ} \text{ mol}^{-1}\right) \left(10^3 \text{ J} \text{ kJ}^{-1}\right) \left(1.602 \times 10^{-19} \text{ J} \text{ eV}^{-1}\right) \left(1 \text{ mol}^{49}\text{Cr}\right) \left(6.022 \times 10^{23} \text{ nuclei} \text{ mol}^{-1}\right)$

$= 2.6270 = 2.6 \text{ MeV}$

24.79 Use the conversion in the chapter ($1 \text{ amu} = 931.5 \text{ MeV}$)
The mass defect is calculated from the mass of 8 protons ($^1\text{H}$) and 8 neutrons versus the mass of the oxygen nuclide.

Mass of 8 $^1\text{H}$ atoms = 8 x 1.007825 = 8.062600 amu
Mass of 8 neutrons = 8 x 1.008665 = 8.069320 amu
Total mass = 16.131920 amu
Mass defect = $\Delta m = 16.131920 - 15.994915 = 0.137005$ amu / $^{16}\text{O} = 0.137005$ g/mol $^{16}\text{O}$

a) Binding energy = $\left(0.137005 \text{ amu}^{16}\text{O}\right) \left(931.5 \text{ MeV} \text{ amu}^{-1}\right)$ = 7.976259844 = 7.976 MeV/nucleon

b) Binding energy = $\left(0.137005 \text{ amu}^{16}\text{O} \text{ atom}^{-1}\right) \left(931.5 \text{ MeV} \text{ amu}^{-1}\right)$ = 127.6201575 = 127.6 MeV/atom

c) Use $\Delta E = \Delta mc^2$

$\text{Binding energy} = \left(0.137005 \text{ g}^{16}\text{O} \text{ mol}^{-1}\right) \left(1 \text{ kg} \text{ g}^{-1}\right) \left(2.99792 \times 10^8 \text{ m} / \text{s}^2\right) \left(\frac{1 \text{ J}}{\text{ kg} \cdot \text{m}^2 / \text{s}^2}\right) \left(\frac{1 \text{ kJ}}{10^3 \text{ J}}\right)$

$= 1.2313357 \times 10^{10} = 1.23134 \times 10^{10} \text{ kJ/mol}$

24.80 The mass defect is calculated from the mass of 82 protons ($^1\text{H}$) and 124 neutrons versus the mass of the lead nuclide.

Mass of 82 $^1\text{H}$ atoms = 82 x 1.007825 = 82.641650 amu
Mass of 124 neutrons = 124 x 1.008665 = 125.074460 amu
Total mass = 207.716110 amu
Mass defect = $\Delta m = 207.716110 - 205.974440 = 1.741670$ amu / $^{206}\text{Pb} = 1.741670$ g/mol $^{206}\text{Pb}$

a) Binding energy = $\left(1.741670 \text{ amu}^{206}\text{Pb} \text{ atom}^{-1}\right) \left(931.5 \text{ MeV} \text{ amu}^{-1}\right)$ = 7.8755611 = 7.876 MeV/atom

b) Binding energy = $\left(1.741670 \text{ amu}^{206}\text{Pb} \text{ atom}^{-1}\right) \left(931.5 \text{ MeV} \text{ amu}^{-1}\right)$ = 1622.3656 = 1622 MeV/atom

c) Binding energy = $\left(1.741670 \text{ g}^{206}\text{Pb} \text{ mol}^{-1}\right) \left(1 \text{ kg} \text{ g}^{-1}\right) \left(2.99792 \times 10^8 \text{ m} / \text{s}^2\right) \left(\frac{1 \text{ J}}{\text{ kg} \cdot \text{m}^2 / \text{s}^2}\right) \left(\frac{1 \text{ kJ}}{10^3 \text{ J}}\right)$

$= 1.5653301 \times 10^{11} = 1.56533 \times 10^{11} \text{ kJ/mol}$
24.81 Calculate $\Delta m$, convert the mass defect to MeV, and divide by 59 nucleons.
Mass of 27 $^1$H atoms = $27 \times 1.007825 = 27.211275$ amu
Mass of 32 neutrons = $32 \times 1.008665 = 32.27728$ amu
Total mass = 59.488555 amu
Mass defect = $\Delta m = 59.488555 - 58.933198 = 0.555357$ amu/59Co = 0.555357 g/mol 59Co

a) Binding energy = $\left( \frac{0.555357 \text{ amu}^{59} \text{Co}}{59 \text{ nucleons}} \right) \left( \frac{931.5 \text{ MeV}}{1 \text{ amu}} \right) = 8.768051619 = 8.768 \text{ MeV/nucleon}$

b) Binding energy = $\left( \frac{0.555357 \text{ amu}^{59} \text{Co}}{1 \text{ atom}} \right) \left( \frac{931.5 \text{ MeV}}{1 \text{ amu}} \right) = 517.3150 = 517.3 \text{ MeV/atom}$

c) Use $\Delta E = \Delta mc^2$

$$\text{Binding energy} = \left( \frac{0.555357 \text{ g}^{59} \text{Co}}{1 \text{ mol}} \right) \left( \frac{1 \text{ kg}}{10^3 \text{ g}} \right) \left( 2.99792 \times 10^8 \text{ m/s} \right)^2 \left( \frac{1 \text{ J}}{\text{ kg \cdot m}^2/\text{s}^2} \right) \left( \frac{1 \text{ kJ}}{10^3 \text{ J}} \right)$$

$$= 4.9912845 \times 10^{10} = 4.99128 \times 10^{10} \text{ kJ/mol}$$

24.82 Calculate $\Delta m$, convert mass defect to MeV, and divide by 131 nucleons.
Mass of 53 $^1$H atoms = $53 \times 1.007825 = 53.414725$ amu
Mass of 78 neutrons = $78 \times 1.008665 = 78.675870$ amu
Total mass = 132.090595 amu
Mass defect = $\Delta m = 132.090595 - 130.906114 = 1.184481$ amu/131I = 1.184481 g/mol 131I

a) Binding energy = $\left( \frac{1.184481 \text{ amu}^{131} \text{I}}{131 \text{ nucleons}} \right) \left( \frac{931.5 \text{ MeV}}{1 \text{ amu}} \right) = 8.422473676 = 8.422 \text{ MeV/nucleon}$

b) Binding energy = $\left( \frac{1.184481 \text{ amu}^{131} \text{I}}{1 \text{ atom}} \right) \left( \frac{931.5 \text{ MeV}}{1 \text{ amu}} \right) = 1103.34405 = 1103 \text{ MeV/atom}$

c) Use $\Delta E = \Delta mc^2$

$$\text{Binding energy} = \left( \frac{1.184481 \text{ g}^{131} \text{I}}{1 \text{ mol}} \right) \left( \frac{1 \text{ kg}}{10^3 \text{ g}} \right) \left( 2.99792 \times 10^8 \text{ m/s} \right)^2 \left( \frac{1 \text{ J}}{\text{ kg \cdot m}^2/\text{s}^2} \right) \left( \frac{1 \text{ kJ}}{10^3 \text{ J}} \right)$$

$$= 1.0645551 \times 10^{11} = 1.06456 \times 10^{11} \text{ kJ/mol}$$

24.83

a) $^{80}{\text{Br}} \rightarrow ^{0}{\text{e}} + ^{76}{\text{Kr}}$ (reaction 1)

$^{80}{\text{Br}} + ^{0}{\text{e}} \rightarrow ^{76}{\text{Se}}$ (reaction 2)

b) Reaction 1: $\Delta m = 79.918528 - 79.916380 = 0.002148$ amu

Reaction 2: $\Delta m = 79.918528 - 79.916520 = 0.002008$ amu

Since $\Delta E = (\Delta m)c^2$, the greater mass change (reaction 1) will release more energy.

24.84 The minimum number of neutrons from each fission event that must be absorbed by the nuclei to sustain the chain reaction is one. In reality, due to neutrons lost from the fissionable material, 2 to 3 neutrons are generally needed to continue a self-sustaining chain reaction.

24.85 In both radioactive decay and fission, radioactive particles are emitted, but the process leading to the emission is different. Radioactive decay is a spontaneous process in which unstable nuclei emit radioactive particles and energy. Fission occurs as the result of high-energy bombardment of nuclei with small particles that cause the nuclides to break into smaller nuclides, radioactive particles, and energy.

In a chain reaction, all fission events are not the same. The collision between the small particle emitted in the fission and the large nucleus can lead to splitting of the large nuclei in a number of ways to produce several different products.
Enriched fissionable fuel is needed in the fuel rods to ensure a sustained chain reaction. Naturally occurring \(^{235}\text{U}\) is only present in a concentration of 0.7%. This is consistently extracted and separated until its concentration is between 3-4%.

Control rods are movable rods of cadmium or boron which are efficient neutron absorbers. In doing so, they regulate the flux of neutrons to keep the reaction chain self-sustaining which prevents the core from overheating.

The moderator is the substance flowing around the fuel and control rods that slows the neutrons, making them better at causing fission.

The reflector is usually a beryllium alloy around the fuel-rod assembly that provides a surface for neutrons that leave the assembly to collide with and therefore, return to the fuel rods.

The water serves to slow the neutrons so that they are better able to cause a fission reaction. Heavy water (\(^{2}\text{H}_2\text{O}\) or \(\text{D}_2\text{O}\)) is a better moderator because it does not absorb neutrons as well as light water (\(^{1}\text{H}_2\text{O}\)) does, so more neutrons are available to initiate the fission process. However, \(\text{D}_2\text{O}\) does not occur naturally in great abundance, so production of \(\text{D}_2\text{O}\) adds to the cost of a heavy water reactor. In addition, if heavy water does absorb a neutron, it becomes tritiated, i.e., it contains the isotope tritium, \(^{3}\text{H}\), which is radioactive.

The advantages of fusion over fission are the simpler starting materials (deuterium and tritium), and no long-lived toxic radionuclide by-products.

Virtually all the elements heavier than helium, up to and including iron, are produced by nuclear fusion reactions in successively deeper and hotter layers of massive stars. Iron is the point at which fusion reactions cease to be energy producers. Elements heavier than iron are produced by a variety of processes, primarily during a supernova event, which distribute the sun’s ash into the cosmos to form next generation suns and planets. Thus, the high cosmic and earth abundance of iron is consistent with it being the most stable of all nuclei.

In the \(s\)-process, a nucleus captures a neutron, emitting a \(\gamma\) ray. Then the nucleus emits a beta particle to form another element. The stable isotopes of most heavy elements form by the \(s\)-process. The \(r\)-process forms less stable isotopes by multiple neutron captures, followed by multiple beta decays.

The unstable \(^{8}\text{Be}\) nucleus forms from a binary collision of \(^{4}\text{He}\) nuclides. Even though the \(^{8}\text{Be}\) is unstable, a certain number exist long enough to capture a third \(^{4}\text{He}\) to form stable \(^{12}\text{C}\). Without this triple \(\alpha\) process, the fusion reaction would stop at helium.

Mass of reactants: 3.01605 + 2.0140 = 5.03005 amu (unrounded)
Mass of products: 4.00260 + 1.008665 = 5.011265 amu (unrounded)
\(\Delta m =\) mass of reactants – mass of products
\(= 5.03005 - 5.011265 = 0.018785\) amu = 0.018785 g/mol (unrounded)
\(\Delta E = \Delta mc^2\)

Energy = \(\left(\frac{0.018785 \text{ g}}{\text{mol}}\right) \left(\frac{1 \text{ kg}}{10^3 \text{ g}}\right) \left(2.99792 \times 10^8 \text{ m/s}\right)^2 \left(\frac{1 \text{ J}}{\text{kg} \cdot \text{m}^2/\text{s}^2}\right) \left(\frac{1 \text{ kJ}}{10^3 \text{ J}}\right)\)

\(= 1.6883064 \times 10^9 = 1.69 \times 10^9\) kJ/mol

\(^{235}\text{U}\) was present from the reaction \(^{243}\text{Am} \rightarrow ^{4}\text{He} + 0\beta + ^{4}\text{He} + ^{235}\text{U}\).

\(^{243}\text{Am} \rightarrow ^{239}\text{Pu} + \frac{3}{2}\text{He}\)

\(\Delta m = \left[243.0614 \text{ amu} - (4.0026 + 239.0522) \text{ amu}\right] \left(\frac{1.66054 \times 10^{-24} \text{ g}}{\text{amu}}\right) \left(\frac{1 \text{ kg}}{10^3 \text{ g}}\right)\)

\(= 1.095956 \times 10^{-29} = 1.1 \times 10^{-29}\) kg
b) \[ E = \Delta mc^2 = \left(1.095956 \times 10^{-29} \text{ kg}\right) \left(2.99792 \times 10^8 \text{ m/s}\right)^2 \left(\frac{1 \text{ J}}{\text{kg} \cdot \text{m}^2/\text{s}^2}\right) = 9.84993 \times 10^{-13} = 9.8 \times 10^{-13} \text{ J} \]

c) \[ E \text{ released} = (9.84993 \times 10^{-13} \text{ J/reaction}) \left(6.022 \times 10^{23} \text{ reactions/mol}\right) \left(1 \text{ kJ} / 10^3 \text{ J}\right) = 5.9316 \times 10^{8} = 5.9 \times 10^{8} \text{ kJ/mol} \]

This is approximately 1 million times larger than a typical heat of reaction.

24.96 a) First, determine the amount of activity released by the $^{239}$Pu for the duration spent in the body (16 h) using the relationship $A = kN$. The rate constant is derived from the half-life and $N$ is calculated using the molar mass and Avogadro’s number.

\[ k = (\ln 2) / t_{1/2} = \left(\frac{\ln 2}{2.41 \times 10^8 \text{ yr}}\right) \left(\frac{1 \text{ yr}}{365.25 \text{ day}}\right) \left(\frac{1 \text{ day}}{24 \text{ h}}\right) \left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 9.113904 \times 10^{-13} \text{ s}^{-1} \text{ (unrounded)} \]

\[ N = (1.00 \mu \text{g Pu}) \left(\frac{10^{-6} \text{ g}}{1 \mu \text{g}}\right) \left(\frac{1 \text{ mol Pu}}{239 \text{ g Pu}}\right) \left(6.022 \times 10^{23} \text{ atoms Pu} \right) \left(1 \text{ mol Pu}\right) = 2.519665 \times 10^{15} \text{ atoms Pu} \text{ (unrounded)} \]

\[ A = kN = (9.113904 \times 10^{-13} \text{ s}^{-1}) \left(2.519665 \times 10^{15} \text{ atoms Pu}\right) = 2.2963987 \times 10^{3} \text{ d/s} \text{ (unrounded)} \]

Each disintegration releases 5.15 MeV, so d/s can be converted to MeV. Convert MeV to J (using 1.602 x 10^{-13} J = 1 MeV) and J to rad (using 0.01 J/kg = 1 rad).

\[ \text{Energy} = \left(\frac{2.2963987 \times 10^{3} \text{ d/s}}{85 \text{ kg}}\right) \left(\frac{5.15 \text{ MeV}}{\text{disint.}}\right) \left(\frac{1.602 \times 10^{-13} \text{ J}}{\text{MeV}}\right) \left(\frac{1 \text{ rad}}{0.01 \text{ J/kg}}\right) \left(\frac{3600 \text{ s}}{1 \text{ h}}\right)\left(16 \text{ h}\right) = 1.2838686 \times 10^{-4} = 1.28 \times 10^{-4} \text{ rads} \]

b) Since 0.01Gy = 1 rad, the worker receives:

\[ \text{Dose} = (1.2838686 \times 10^{-4} \text{ rad}) \left(0.01 \text{ Gy/rad}\right) = 1.2838686 \times 10^{-6} = 1.28 \times 10^{-6} \text{ Gy} \]

24.97 Determine $k$ for $^{14}$C using the half-life (5730 yr):

\[ k = (\ln 2) / t_{1/2} = (\ln 2) / (5730 \text{ yr}) = 1.2096809 \times 10^{-4} \text{ yr}^{-1} \text{ (unrounded)} \]

Determine the mass of carbon in 4.58 grams of CaCO$_3$:

\[ \text{mass} = \left(4.58 \text{ g CaCO}_3\right) \left(\frac{1 \text{ mol CaCO}_3}{100.09 \text{ g CaCO}_3}\right) \left(\frac{1 \text{ mol C}}{1 \text{ mol CaCO}_3}\right) \left(\frac{12.01 \text{ g C}}{1 \text{ mol C}}\right) = 0.5495634 \text{ g C} \text{ (unrounded)} \]

The activity is: (3.2 dpm) / (0.5495634 g C) = 5.8228 dpm g$^{-1}$

Using the integrated rate law:

\[ \ln \left(\frac{N_t}{N_0}\right) = -kt \quad N_0 = 15.3 \text{ dpm g}^{-1} \text{ (the ratio of } ^{12}\text{C}:{^{14}\text{C} \text{ in living organisms)})} \]

\[ \ln \left(\frac{5.8228 \text{ dpm g}^{-1}}{15.3 \text{ dpm g}^{-1}}\right) = -\left(1.2096809 \times 10^{-4} \text{ yr}^{-1}\right)t \]

\[ t = 7986.17 = 8.0 \times 10^{3} \text{ yr} \]

24.98 Find the rate constant from the rate of decay and the initial number of atoms. Use rate constant to calculate half-life.

Initial number of atoms:

\[ \text{Ra atoms} = (5.4 \mu \text{gRaCl}_2) \left(\frac{10^{-6} \text{ g}}{1 \mu \text{g}}\right) \left(\frac{1 \text{ mol RaCl}_2}{297 \text{ g RaCl}_2}\right) \left(\frac{1 \text{ mol Ra}}{1 \text{ mol RaCl}_2}\right) \left(\frac{6.022 \times 10^{23} \text{ Ra atoms}}{1 \text{ mol Ra}}\right) = 1.102909 \times 11^{16} \text{ Ra atoms (unrounded)} \]
\[ k = \left( \frac{1.5 \times 10^5 \text{ Bq}}{1.102909 \times 10^{16} \text{ Ra atoms}} \right) \left( \frac{1 \text{ d}/\text{s}}{\text{Bq}} \right) = 1.3600395 \times 10^{-11} \text{ s}^{-1} \text{ (unrounded)} \]

\[ t_{1/2} = (\ln 2) / k = (\ln 2) / (1.3600395 \times 10^{-11} \text{ s}^{-1}) = 5.0965 \times 10^{10} = 5.1 \times 10^{10} \text{ s} \]

\[ k = (\ln 2) / t_{1/2} = (\ln 2) / (5730 \text{ yr}) = 1.20968 \times 10^{-4} \text{ yr}^{-1} \text{ (unrounded)} \]

\[ \text{Number of atoms} = (14 \times 23 \times 14) / (14 \text{ g C} / 1 \text{ mol C}) \]

\[ = 4.3014 \times 10^{14} \text{ atoms } ^{14}\text{C} \text{ (unrounded)} \]

\[ A = kN = [(1.20968 \times 10^{-4} \text{ yr}^{-1}) (4.3014 \times 10^{14} \text{ atoms } ^{14}\text{C})] / (1 \text{ disintegration} / 1 \text{ atom}) = 5.2033 \times 10^{10} \text{ dpyr} \]

\[ \text{Dose} = \left( \frac{5.2033 \times 10^{10} \text{ dpyr}}{65 \text{ kg}} \right) \left( \frac{0.156 \text{ MeV}}{\text{disint.}} \right) \left( \frac{1.602 \times 10^{-13} \text{ J}}{1 \text{ MeV}} \right) \left( \frac{1 \text{ rad}}{0.01 \text{ J/kg}} \right) = 2.097 \times 10^{-3} = 10^{-3} \text{ rad} \]

\[ \Delta m = \text{mass of reactants} – \text{mass of products} \]

\[ = [(4) (1.007825)] – [(4.00260 + (2) (5.48580 \times 10^{-4})] \]

\[ = 4.031300 – 4.002697 = 0.02760 \text{ g/mol } ^{4}\text{He} \]

\[ \text{Energy} = \left( \frac{0.02760 \text{ amu } ^{4}\text{He}}{1 \text{ atom}} \right) \left( \frac{931.5 \text{ MeV}}{1 \text{ amu}} \right) = 25.7094 = 25.71 \text{ MeV / atom} \]

\[ \text{Convert atoms to moles using Avogadro’s number.} \]

\[ \text{Energy} = (25.7094 \text{ MeV / atom}) (6.022 \times 10^{23} \text{ atom/mol}) = 1.54822 \times 10^{25} = 1.548 \times 10^{25} \text{ MeV/mol} \]

\[ \text{Determine how many grams of AgCl are dissolved in 1 mL of solution. The activity of the radioactive Ag}^{+} \text{indicates how much AgCl dissolved, given a starting sample with a specific activity (175 nCi/g).} \]

\[ \text{Concentration} = \left( \frac{1.25 \times 10^{-2} \text{ Bq}}{\text{mL}} \right) \left( \frac{1 \text{ dps}}{1 \text{ Bq}} \right) \left( \frac{1 \text{ Ci}}{3.70 \times 10^{10} \text{ dps}} \right) \left( \frac{1 \text{ nCi}}{1 \text{ Ci}} \right) \left( \frac{1 \text{ g AgCl}}{175 \text{ nCi}} \right) \]

\[ = 1.93050 \times 10^{-6} \text{ g AgCl/mL} \text{ (unrounded)} \]

\[ \text{Convert g/mL to mol/L (molar solubility) using the molar mass of AgCl.} \]

\[ \text{Molarity} = \left( \frac{1.93050 \times 10^{-6} \text{ g AgCl}}{\text{mL}} \right) \left( \frac{1 \text{ mol AgCl}}{143.4 \text{ g AgCl}} \right) \left( \frac{1 \text{ mL}}{10^{-3} \text{ L}} \right) \]

\[ = 1.34623 \times 10^{-5} = 1.35 \times 10^{-5} \text{ M AgCl} \]

\[ \text{The process shown is fission in which a neutron bombards a large nucleus, splitting that nucleus into two nuclei of intermediate mass.} \]

\[ _{92}^{235}\text{U} + {\text{n}} \rightarrow {^{144}\text{Cs}} + {^{90}\text{Rb}} + 2_{0}^{1}\text{n} \]

\[ {^{144}\text{Cs}} , \text{with 55 protons and 89 neutrons, has a n/p ratio of 1.6. This ratio places this isotope above the band of stability and decay by beta particle emission is expected.} \]

\[ \text{Determine the value of } k \text{ from the half-life. Then determine the fraction from the integrated rate law.} \]

\[ k = (\ln 2) / t_{1/2} = (\ln 2) / (7.0 \times 10^{8} \text{ yr}) = 9.90210 \times 10^{-10} \text{ yr}^{-1} \text{ (unrounded)} \]

\[ \ln \left( \frac{N_{t}}{N_{0}} \right) = -kt = (9.90210 \times 10^{-10} \text{ yr}^{-1}) (2.8 \times 10^{9} \text{ yr}) = -2.772588 \text{ (unrounded)} \]

\[ \left( \frac{N_{t}}{N_{0}} \right) = 0.062500 = 6.2 \times 10^{-2} \]
24.104 Determine the value of \( k \) from the half-life. Then determine the age from the integrated rate law.

\[
k = \frac{(\ln 2)}{t_{1/2}} = \frac{(\ln 2)}{(4.5 \times 10^9 \text{ yr})} = 1.540327 \times 10^{-10} \text{ yr}^{-1} \text{ (unrounded)}
\]

\[
\ln \left( \frac{N_t}{N_0} \right) = -kt
\]

\[
\ln \left( \frac{6}{6 + 9} \right) = - (1.540327 \times 10^{-10} \text{ yr}^{-1}) (t)
\]

\[
-0.916291 = - (1.540327 \times 10^{-10} \text{ yr}^{-1}) (t)
\]

\[
t = 5.948677 \times 10^9 = 5.9 \times 10^9 \text{ yr}
\]

24.105 a) The simultaneous fusion of three nuclei is a termolecular process. Termolecular processes have a very low probability of occurring. The bimolecular fusion of \(^8\text{Be}\) with \(^4\text{He}\) is more likely.

24.106 a) Find the rate constant, \( k \), using any two data pairs (the greater the time between the data points, the greater the reliability of the calculation). Calculate \( t_{1/2} \) using \( k \).

\[
\ln \left( \frac{N_t}{N_0} \right) = -kt
\]

\[
\ln \left( \frac{\frac{495 \text{ photons/s}}{5000 \text{ photons/s}}}{} \right) = -(0.11563 \text{ h}^{-1}) (2.0 \text{ h}) = -0.23126 \text{(unrounded)}
\]

\[
k = 0.11563 \text{ h}^{-1} \text{ (unrounded)}
\]

\[
t_{1/2} = (\ln 2) / k = (\ln 2) / (0.11563 \text{ h}^{-1}) = 5.9945 = 5.99 \text{ h} \text{ (Assuming the times are exact, and the emissions have three significant figures.)}
\]

b) The percentage of isotope remaining is the fraction remaining after 2.0 h (\( N_t \) where \( t = 2.0 \text{ h} \)) divided by the initial amount (\( N_0 \)), i.e., fraction remaining is \( N_t / N_0 \). Solve the first order rate expression for \( N_t / N_0 \), and then subtract from 100% to get fraction lost.

\[
\ln \left( \frac{N_t}{N_0} \right) = -kt
\]

\[
\ln \left( \frac{N_t}{N_0} \right) = -(0.11563 \text{ h}^{-1}) (2.0 \text{ h}) = -0.23126 \text{(unrounded)}
\]

\[
\left( \frac{N_t}{N_0} \right) \times 100\% = 79.3533\% \text{(unrounded)}
\]

The fraction lost is 100% – 79.3533% = 20.6467% = 21% of the isotope is lost upon preparation.

24.107 \( k = (\ln 2) / t_{1/2} = (\ln 2) / (1.25 \times 10^9 \text{ yr}) = 5.5451774 \times 10^{-10} \text{ yr}^{-1} \text{ (unrounded)} \)

\( A = kN \) where \( A = \text{dps} \) \( N = \text{number of atoms} \)

Atoms = (1.0 mol \(^{40}\text{K}\)) (6.022 x \(10^{23}\) atoms/mol) = 6.022 x \(10^{23}\) atoms \(^{40}\text{K}\) (unrounded)

\[
A = \left( \frac{5.5451774 \times 10^{-10}}{\text{yr}} \right) (6.022 \times 10^{23} \text{ atoms}) \left( 1 \text{ yr} \right) \left( 365.25 \text{ day} \right) \left( 1 \text{ disint.} \right) \left( 1 \text{ h} \right) \left( 3600 \text{ s} \right) \left( 1 \text{ atom} \right)
\]

= 1.05816 \times 10^7 \text{ dps} \text{ (unrounded)}

Dose = (1.05816 \times 10^7 \text{ dps}) (1 Ci / 3.70 \times 10^{10} \text{ dps}) = 2.85989 \times 10^{-4} = 2.9 \times 10^{-4} \text{ Ci}

Dose = (1.05816 \times 10^7 \text{ dps}) (1 Bq / 1 \text{ dps}) = 1.05816 \times 10^7 = 1.1 \times 10^7 \text{ Bq}

24.108 a) fraction remaining after 10.0 yr = \( \left( \frac{1}{2} \right)^{10/\sqrt{2}} = \left( \frac{1}{2} \right)^{10/5730} = 0.998791 = 0.999 \)

b) fraction remaining after 10.0 \times 10^3 \text{ yr} = \( \left( \frac{1}{2} \right)^{10 \times 10^3/5730} = 0.298292 = 0.298 \)
c) Fraction remaining after $10.0 \times 10^4$ yr = \left( \frac{1}{2} \right)^{\frac{10.0 \times 10^4}{5730}} = 5.577295 \times 10^{-6} = 5.58 \times 10^{-6}$

d) Radiocarbon dating is more reliable for (b) because a significant quantity of $^{14}$C has decayed and a significant quantity remains. Therefore, a change in the amount of $^{14}$C would be noticeable. For the fraction in (a), very little $^{14}$C has decayed and for (c) very little $^{14}$C remains. In either case, it will be more difficult to measure the change so the error will be relatively large.

24.109 $^{210}$Rn $+ \, _{0}^{+}e \rightarrow ^{210}$At
Mass = (2.368 MeV) (1 amu / 931.5 MeV) = 0.0025421363 amu (unrounded)
Mass $^{210}$At = mass $^{210}$Rn + electron mass – mass equivalent of energy emitted.
= (209.989669 + 0.000549 – 0.0025421363) amu
= 209.9876759 = 209.98768 amu

24.110 At 1 half-life, the fraction of sample is 0.500.
$(0.900)^n = 0.500$
$n \ln(0.900) = \ln(0.500)$
$n = (\ln(0.500)) / (\ln(0.900)) = 6.58$ h

24.111 a) $^5_{23}$V $+ \, _{0}^{+}n \rightarrow ^{52}_{23}$V $\rightarrow ^{52}_{24}$Cr $+$ $^0_{-1}\beta$
$^{51}_{23}$V (n,$\beta$)$^{52}_{24}$Cr
b) Positron emission by copper-64 produces nickel-64.
$^{63}_{29}$Cu $+ \, _{0}^{+}n \rightarrow ^{64}_{29}$Cu $\rightarrow ^{64}_{28}$Ni $+$ $^0_{+1}\beta$
$^{63}_{29}$Cu (n,$\beta$)$^{64}_{28}$Ni
c) $^\beta$ decay by aluminum-28 produces silicon-28.
$^{27}_{13}$Al $+ \, _{0}^{+}n \rightarrow ^{28}_{13}$Al $\rightarrow ^{28}_{14}$Si $+$ $^0_{-1}\beta$
$^{27}_{13}$Al (n,$\beta$)$^{28}_{14}$Si

24.112 Determine $k$ for $^{90}$Sr:
$$k = (\ln k) / t_{1/2} = (\ln 2) / 29 \, \text{yr} = 0.02390 \, \text{yr}^{-1} \text{ (unrounded)}$$

a) \[ \ln \left( \frac{N_t}{N_0} \right) = -kt \]
\[ \ln \left( \frac{N_{0.0500 \, g}}{0.0500 \, g} \right) = -(0.02390 \, \text{yr}^{-1}) (10 \, \text{yr}) = -0.2390 \text{ (unrounded)} \]
$$\left( \frac{N_{0.0500 \, g}}{0.0500 \, g} \right) = 0.78741 \text{ (unrounded)}$$
$$0.78741 (0.0500 \, g) = 0.0393707 = 0.039 \, g^{90}\text{Sr}$$

b) \[ \ln \left( \frac{N_t}{N_0} \right) = -kt \]
\[ \ln \left( \frac{(100 - 99.9) \, \%}{100 \%} \right) = -(0.02390 \, \text{yr}^{-1}) t \]
$$t = 289.027 = 3 \times 10^2 \, \text{yr} \text{ (The calculation 100 – 99.9 limits the answer to one significant figure.)}$$

24.113 a) $^{12}_{6}$C $+ \, ^{4}_{2}\text{He} \rightarrow ^{16}_{8}\text{O}$
$$b) \left(7.7 \times 10^{-2} \, \text{amu}\right) \left(931.5 \, \text{MeV} / 1 \, \text{amu}\right) \left(1.602 \times 10^{-13} \, \text{J} / 1 \, \text{MeV}\right) \left(1 \, \text{kJ} / 10^3 \, \text{J}\right) = 1.1490425 \times 10^{-14} = 1.1 \times 10^{-14} \, \text{kJ}$$
24.114 The production rate of radon gas (volume/hour) is also the decay rate of $^{226}\text{Ra}$. The decay rate, or activity, is proportional to the number of radioactive nuclei decaying, or the number of atoms in 1.000 g of $^{226}\text{Ra}$, using the relationship $A = kN$. Calculate the number of atoms in the sample, and find $k$ from the half-life. Convert the activity in units of nuclei/time (also disintegrations per unit time) to volume/time using the ideal gas law.

$^{226}\text{Ra} \rightarrow ^4\text{He} + ^{222}\text{Rn}$

$k = (\ln 2) / t_{1/2} = (\ln 2) / [(1599 \text{ yr}) (8766 \text{ h/yr})] = 4.9451051 \times 10^{-8} \text{ h}^{-1}$ (unrounded)

The mass of $^{226}\text{Ra}$ is 226.025402 amu/atom or 226.025402 g/mol.

$$N = \left( \frac{1 \text{ mol Ra}}{226.025402 \text{ g Ra}} \right) \left( \frac{6.022 \times 10^{23} \text{ Ra atoms}}{1 \text{ mol Ra}} \right) = 2.6643023 \times 10^{21} \text{ Ra atoms}$$ (unrounded)

$$A = kN = (4.9451051 \times 10^{-8} \text{ h}^{-1}) (2.6643023 \times 10^{21} \text{ Ra atoms}) = 1.317525 \times 10^{14} \text{ Ra atoms/h}$$ (unrounded)

This result means that 1.318 x $10^{14} ^{226}\text{Ra}$ nuclei are decaying into $^{222}\text{Rn}$ nuclei every hour. Convert atoms of $^{222}\text{Rn}$ into volume of gas using the ideal gas law.

$$V = \frac{nRT}{P} = \left( \frac{2.1878536 \times 10^{-10} \text{ mol Rn/h}}{0.08206 \text{ L} \cdot \text{atm} / \text{mol} \cdot \text{K}} \right) \left( \frac{273.15 \text{ K}}{1 \text{ atm}} \right) = 4.904006 \times 10^{-9} = 4.904 \times 10^{-9} \text{ L/h}$$

Therefore, radon gas is produced at a rate of $4.904 \times 10^{-9}$ L/h. Note: Activity could have been calculated as decay in moles/time, removing Avogadro’s number as a multiplication and division factor in the calculation.

24.115 Determine $k$:

$$k = (\ln 2) / t_{1/2} = (\ln 2) / (32 \text{ s}) = 0.0216608 \text{ s}^{-1}$$ (unrounded)

$$\ln \left( \frac{N_t}{N_0} \right) = -kt$$

$$\ln \left( \frac{90\%}{100\%} \right) = -(0.0216608 \text{ s}^{-1})t$$

$$t = 4.86411 = 4.9 \text{ s}$$

24.116 a) $^{133}\text{Cs}$ The N/Z ratio for $^{140}\text{Cs}$ is too high.

b) $^{79}\text{Br}$ It has an even number of neutrons compared with $^{78}\text{Br}$.

c) $^{24}\text{Mg}$ The N/Z ratio equals 1.

d) $^{14}\text{N}$ The N/Z ratio equals 1.

24.117 Determine $k$ from the half-life

$$k = (\ln 2) / t_{1/2} = (\ln 2) / (29 \text{ yr}) = 0.023902 \text{ yr}^{-1}$$ (unrounded)

$$\ln \left( \frac{N_t}{N_0} \right) = -kt$$

$$\ln \left( \frac{1.0 \times 10^4 \text{ particles}}{7.0 \times 10^4 \text{ particles}} \right) = -(0.023902 \text{ yr}^{-1})t$$

$$-1.945910 = -(0.023902 \text{ yr}^{-1})t$$

$$t = 81.412022 = 81 \text{ yr}$$

24-18
24.118  a) $^{6}\text{Li} + ^{3}\text{Li} \rightarrow ^{12}\text{C}$ (dilithium)

b) $\Delta m = 2 \text{ (mass } ^{6}\text{Li) } - \text{ mass } ^{12}\text{C}$

$$\Delta m = (0.030242 \text{ amu/atom}) (1.66054 \times 10^{-27} \text{ kg/amu}) = 5.021805 \times 10^{-29} \text{ kg/atom}$$

$$E = \Delta mc^2 = \left( 5.021805 \times 10^{-29} \text{ kg/atom} \right) \left( \left( 2.99792 \times 10^8 \text{ m/s} \right)^2 \right) = 4.5133595 \times 10^{-12} \text{ J/atom}$$

$$E = \left( 4.5133595 \times 10^{-12} \text{ J/atom} \right) \left( \frac{1 \text{ atm}}{12.000000 \text{ amu}} \right) \left( \frac{1 \text{ amu}}{1.66054 \times 10^{-27} \text{ kg}} \right) = 2.2650059 \times 10^{-14} = 2.2650 \times 10^{14} \text{ J/kg dilithium}$$

c) $^{4}\text{H} \rightarrow ^{4}\text{He} + 2^0\beta$ 2 positrons are released.

d) For Dilithium ($^{12}\text{C}$):

Mass = (5.021805 x 10^{-29} kg/atom) (1 atom / 12.000000 amu) (1 amu / 1.66054 x 10^{-27} kg 12C)

$$= 2.5202892 \times 10^{-3} = 2.5202 \times 10^{-3} \text{ kg / kg } ^{12}\text{C}$$

For $^{4}\text{He}$ (The mass of a positron is the same a the mass of an electron.)

$$\Delta m = 4 \text{ mass } ^{1}\text{H} - [\text{ mass } ^{4}\text{He} + \text{ 2 mass } _{0}\text{e}]$$

$$= 4 (1.007825 \text{ amu}) - [4.00260 \text{ amu} + 2(5.48580 \times 10^{-4} \text{ amu})]$$

$$= 4.031300 \text{ amu} - 4.00370 \text{ amu}$$

$$= 0.02760 \text{ amu/atom}$$

$$\Delta m = (0.02760 \text{ amu/atom}) (1.66054 \times 10^{-27} \text{ kg/amu}) = 4.58309 \times 10^{-29} \text{ kg/atom}$$

$$\text{Mass} = (4.58309 \times 10^{-29} \text{ kg/atom}) (1 \text{ amu} / 1.66054 \times 10^{-27} \text{ kg } ^{4}\text{He})$$

$$= 6.895517 \times 10^{-3} = 6.896 \times 10^{-3} \text{ kg / kg } ^{4}\text{He}$$

e) $^{3}\text{H} + ^{1}\text{H} \rightarrow ^{4}\text{He} + ^{0}\text{n}$

$$\Delta m = [2.0140 \text{ amu} + 3.01605 \text{ amu}] - [4.00260 \text{ amu} + 1.008665 \text{ amu}]$$

$$= 5.0300 \text{ amu} - 5.01126 \text{ amu} = 0.0188 \text{ amu}$$

$$= (0.0188 \text{ amu/atom}) (1.66054 \times 10^{-27} \text{ kg/amu}) = 3.121852 \times 10^{-29} \text{ kg/atom}$$

$$\text{Mass} = (3.121852 \times 10^{-29} \text{ kg/atom}) (1 \text{ amu} / 1.66054 \times 10^{-27} \text{ kg } ^{4}\text{He})$$

$$= 4.6969469 \times 10^{-3} = 4.70 \times 10^{-3} \text{ kg / kg } ^{4}\text{He}$$

f) $^{6}\text{Li} + _{0}\text{n} \rightarrow ^{4}\text{He} + ^{1}\text{H}$

$$\text{Mass } ^{3}\text{H} / \text{ kg } ^{4}\text{He} = \left( \frac{1 \text{ mol } ^{3}\text{H}}{1 \text{ mol } ^{6}\text{Li}} \right) \left( \frac{3.01605 \text{ g } ^{3}\text{H}}{1 \text{ mol } ^{3}\text{H}} \right) \left( \frac{1 \text{ mol } ^{6}\text{Li}}{1 \text{ mol } ^{6}\text{Li}} \right) \left( \frac{10^3 \text{ g } ^{6}\text{Li}}{1 \text{ kg } ^{6}\text{Li}} \right) \left( \frac{1 \text{ kg } ^{7}\text{H}}{10^3 \text{ g } ^{3}\text{H}} \right)$$

$$= 0.501411359 = 0.501411 \text{ kg } ^{3}\text{H} / \text{kg } ^{6}\text{Li}$$

$$\text{Mass} = (0.501411359 \text{ g } ^{3}\text{H}) \left( \frac{1 \text{ mol } ^{3}\text{H}}{3.01605 \text{ g } ^{3}\text{H}} \right) \left( \frac{10^3 \text{ g } ^{6}\text{Li}}{1 \text{ mol } ^{6}\text{Li}} \right) \left( \frac{6.022 \times 10^{23} \text{ atom } ^{3}\text{H}}{1 \text{ mol } ^{3}\text{H}} \right) \left( \frac{3.121852 \times 10^{29} \text{ g } ^{3}\text{H}}{1 \text{ atom } ^{3}\text{H}} \right)$$

$$= 3.1254222 \times 10^{-3} = 3.125 \times 10^{-3} \text{ kg}$$

Mass defect for dilithium reaction:

Mass = (5.021805 x 10^{-29} kg / atom 12C) (1 atom 12C / 2 atoms 6Li) (6.022 x 10^{23} atoms 6Li / mol) (1 mol 6Li / 6.015121 g 6Li) (10^3 g / kg 6Li) = 2.513774 x 10^{-3} = 2.514 x 10^{-3} kg

The mass defect for the dilithium reaction is slightly less than the mass defect for the fusion of tritium with deuterium.

24.119  a) Convert pCi to Bq using the conversion factors 1 Ci = 3.70 x 10^{10} Bq and 1 pCi = 10^{-12} Ci.

Activity = \left( \frac{4.0 \text{ pCi}}{1 \text{ Ci}} \right) \left( \frac{10^{-12} \text{ Ci}}{1 \text{ pCi}} \right) \left( \frac{3.70 \times 10^{10} \text{ Bq}}{1 \text{ Ci}} \right) = 0.148 = 0.15 \text{ Bq/L}

The safe level is 0.15 Bq/L.
b) Use the first-order rate expression to find the activity at the later time \( t = 9.5 \) days.

\[
k = \frac{(\ln 2)}{3.82 \text{ days}} = 0.181452 \text{ days}^{-1} \quad \text{(unrounded)}
\]

\[
\ln \left( \frac{N_t}{N_0} \right) = -kt
\]

\[
\ln \left( \frac{41.5 \text{ pCi/L}}{N_t} \right) = -(0.181452 \text{ days}^{-1})(9.5 \text{ days}) = -1.723794 \quad \text{(unrounded)}
\]

\[
\frac{N_t}{41.5 \text{ pCi/L}} = 0.17838806 \quad \text{(unrounded)}
\]

\[
N_t = 7.403104 \text{ pCi/L}
\]

Activity = \[
\frac{7.403104 \text{ pCi/L} \times 10^{-12} \text{ Ci/L} \times 3.70 \times 10^{10} \text{ Bq/Ci}}{1 \text{ Ci}} = 0.2739148 = 0.27 \text{ Bq/L}
\]

c) The desired activity is 0.15 Bq/L, however, the room air currently contains 0.27 Bq/L. Rearrange the first-order rate expression to solve for time.

\[
\ln \left( \frac{N_t}{N_0} \right) = -kt
\]

\[
\ln \left( \frac{0.15 \text{ Bq/L}}{0.2739148 \text{ Bq/L}} \right) = -(0.181452 \text{ days}^{-1})t
\]

\[-0.602182 = -(0.181452 \text{ days}^{-1})t
\]

\[t = 3.318685 = 3.3 \text{ days}\]

It takes 3.3 days more to reach the recommended EPA level. A total of 12.8 (3.3 + 9.5) days is required to reach recommended levels when the room was initially measured at 41.5 pCi/L.

24.120 \[t_{1/2} = \frac{(\ln 2)}{k} = \frac{(\ln 2)}{12.26 \text{ yr}} = 0.05653729 \text{ yr}^{-1} \quad \text{(unrounded)}\]

\[
\ln \left( \frac{N_t}{N_0} \right) = -kt
\]

\[
\ln \left( \frac{N_t}{N_0} \right) = -(0.05653729 \text{ yr}^{-1})(5.50 \text{ yr}) = -0.310955097 \quad \text{(unrounded)}
\]

\[
\frac{N_t}{N_0} = 0.732746777 \quad \text{(unrounded)}
\]

Fraction lost = \[1 - 0.732746777 = 0.267253222 = 0.267\]

24.121 \[
^{239}\text{U} \rightarrow \beta^- + ^{239}\text{Np} \rightarrow ^{239}\text{Pu} \rightarrow \beta^- + ^{235}\text{Pu} \rightarrow \alpha + ^{235}\text{U} \rightarrow \alpha + ^{231}\text{Th}
\]

This could begin the \(^{235}\text{U}\) decay series.

24.122 The energy per time is calculated from the disintegrations per time (1.0 mCi) and the energy per disintegration (5.59 MeV).

\[
(1.0 \text{ mCi}) \left( \frac{10^{-3} \text{ Ci}}{1 \text{ mCi}} \right) \left( \frac{3.70 \times 10^{10} \text{ dps}}{1 \text{ Ci}} \right) \left( \frac{5.59 \text{ MeV}}{1 \text{ disint.}} \right) \left( \frac{1.602 \times 10^{-13} \text{ J}}{1 \text{ MeV}} \right) = 3.3134166 \times 10^{-5} \text{ J/s} \quad \text{(unrounded)}
\]

Convert the mass of the child to kg:

\[
(54 \text{ lb}) \left( \frac{1 \text{ kg}}{2.205 \text{ lb}} \right) = 24.4897959 \text{ kg} \quad \text{(unrounded)}
\]
Time for 1.0 mrad to be absorbed:

\[
\text{Time} = (1.0 \text{ mrad}) \left( \frac{10^{-3} \text{ rad}}{1 \text{ mrad}} \right) \left( \frac{0.01 \text{ J/kg}}{1 \text{ rad}} \right) \left( \frac{24.49 \text{ kg}}{3.31 \times 10^{16} \times 10^{-5} \text{ J/s}} \right) = 7.3911007 = 7.4 \text{ s}
\]

24.123 \( k = (\ln 2) / t_{1/2} = (\ln 2) / 5730 \text{ yr} = 1.2096809 \times 10^{-4} \text{ yr}^{-1} \) (unrounded)

\[
\ln \left( \frac{N_t}{N_0} \right) = -kt
\]

\[
\ln \left( \frac{12.9 \text{ d/min \cdot g}}{15.3 \text{ d/min \cdot g}} \right) = -(1.2096809 \times 10^{-4} \text{ yr}^{-1})t
\]

\[-0.170625 = -(1.2096809 \times 10^{-4} \text{ yr}^{-1})t\]

\[t = 1321.686557 = 1.32 \times 10^3 \text{ yr}\]

The earthquake occurred 1320 years ago.

24.124 Assuming our atmosphere present today was not fully developed and the fact that much of the cosmic ionizing radiation is absorbed by the atmosphere, organisms would have been exposed to more cosmic radiation a billion years ago. In addition, the number of radioactive nuclei such as uranium would have been larger. This would also have the effect of exposing organisms to greater ionizing radiation from the earth.

24.125 Because the 1941 wine has a little over twice as much tritium in it, just over one half-life has passed between the two wines. Therefore, the older wine was produced before 1929 (1941 – 12.26) but not much earlier than that. To find the number of years back in time, use the first order rate expression, where \( N_0 = 2.32 \text{ N}, N_t = N \) and \( t = \) years transpired between the manufacture date and 1941.

\[
k = (\ln 2) / t_{1/2} = (\ln 2) / 12.26 \text{ yr} = 0.05653729 \text{ yr}^{-1} \) (unrounded)
\]

\[
\ln \left( \frac{N_t}{N_0} \right) = -kt
\]

\[
\ln \left( \frac{N}{2.32 \text{ N}} \right) = -(0.05653729 \text{ yr}^{-1})t
\]

\[-0.841567 = -(0.05653729 \text{ yr}^{-1})t\]

\[t = 14.8851669 = 14.9 \text{ yr}\]

The wine was produced in (1941 – 15) = 1926.

24.126 If 99% of Pu-239 decays, 1% remains.

\[
k = (\ln 2) / t_{1/2} = (\ln 2) / 2.41 \times 10^4 \text{ yr} = 2.8761293 \times 10^{-5} \text{ yr}^{-1} \) (unrounded)
\]

\[
\ln \left( \frac{N_t}{N_0} \right) = -kt
\]

\[
\ln \left( \frac{1\%}{100\%} \right) = -(2.8761293 \times 10^{-5} \text{ yr}^{-1})t
\]

\[-4.60517 = -(2.8761293 \times 10^{-5} \text{ yr}^{-1})t\]

\[t = 1.6011693 \times 10^5 = 2 \times 10^5 \text{ yr}\]

(The 1% limits the answer to one significant figure.)

24.127 \( k = (\ln 2) / t_{1/2} = (\ln 2) / 5730 \text{ yr} = 1.2096809 \times 10^{-4} \text{ yr}^{-1} \) (unrounded)

\[
\ln \left( \frac{N_t}{N_0} \right) = -kt
\]

\[
\ln \left( \frac{0.61 \text{ pCi/g}}{6.89 \text{ pCi/g}} \right) = -(1.2096809 \times 10^{-4} \text{ yr}^{-1})t
\]

\[-2.4243674 = -(1.2096809 \times 10^{-4} \text{ yr}^{-1})t\]

\[t = 2.0041 \times 10^4 = 2.0 \times 10^4 \text{ yr}\]
24.128 Determine the mass defect for the reaction:

\[
\Delta m = \text{mass of reactants} - \text{mass of products} = \left( 14.003074 + 1.008665 \right) - \left( 14.003241 + 1.007825 \right) = 15.011739 - 15.011066 = 0.000673 \text{ amu}
\]

Energy = \( (0.000673 \text{ amu}) \left( \frac{931.5 \text{ MeV}}{1 \text{ amu}} \right) \left( \frac{10^6 \text{ eV}}{1 \text{ MeV}} \right) = 6.268995 \times 10^5 \text{ eV} \)

Energy = \( (6.268995 \times 10^5 \text{ eV}) \left( \frac{6931.5 \text{ MeV}}{10^6 \text{ eV}} \right) \left( \frac{1 \text{ amu}}{1 \text{ atom}} \right) = 6.0478524 \times 10^7 = 6.05 \times 10^7 \text{ kJ/mol} \)

24.129 Calculate \( \Delta m \).

Mass of 3 \( ^1 \text{H} \) atoms = \( 3 \times 1.007825 = 3.023475 \text{ amu} \)

Mass of 4 neutrons = \( 4 \times 1.008665 = 4.034660 \text{ amu} \)

Total mass = 7.058135 amu

Mass defect = \( \Delta m = 7.058153 - 7.016003 = 0.042132 \text{ g/mol} \)

Binding energy = \( 760.042132 \text{ amu} \left( \frac{7\text{Li}}{1 \text{ atom}} \right) \left( \frac{931.5 \text{ MeV}}{1 \text{ amu}} \right) \left( \frac{1 \text{ amu}}{1 \text{ ato m}} \right) \left( \frac{1 \text{ MeV}}{1 \text{ eV}} \right) \)

\( = 3.9245958 \times 10^7 = 3.925 \times 10^7 \text{ eV/nucleus} \)

Convert MeV to kJ.

\( \text{ Binding energy} = \left( \frac{0.042132}{1 \text{ mol}} \right) \left( \frac{7\text{Li}}{1 \text{ atom}} \right) \left( \frac{931.5 \text{ MeV}}{1 \text{ amu}} \right) \left( \frac{10^6 \text{ eV}}{1 \text{ MeV}} \right) \left( \frac{1 \text{ kg}}{1 \text{ atm}} \right) \left( \frac{1 \text{ mol}}{1 \text{ H atm}} \right) \left( \frac{1 \text{ m/s}}{1 \text{ kg}} \right) \left( \frac{1 \text{ J}}{1 \text{ kg} \cdot \text{m}^2/\text{s}^2} \right) \left( \frac{1 \text{ kJ}}{1 \text{ mol H}} \right) \)

\( = 3.7866237 \times 10^9 = 3.786 \times 10^9 \text{ kJ/mol} \)

24.130 \( ^{146}_{64} \text{Gd} + ^0_{-1} \text{e} \rightarrow ^{146}_{63} \text{Eu} \rightarrow ^{146}_{62} \text{Sm} + ^0_1 \beta \rightarrow ^2_4 \text{He} + ^{142}_{60} \text{Nd} \)

24.131 a) Kinetic energy = \( \frac{1}{2} m v^2 = (1/2) m (3 RT / N_A) = (3/2) (RT / N_A) \)

Energy = \( \left( \frac{3}{2} \right) \left( \frac{8.314 \text{ J/mol} \cdot \text{K}}{1 \text{ K}} \right) \left( \frac{1.00 \times 10^8 \text{ K}}{1 \text{ K}} \right) \)

\( = 2.0709066 \times 10^{-17} = 2.07 \times 10^{-17} \text{ J/atom} \)

b) A kilogram of \( ^1 \text{H} \) will annihilate a kilogram of anti-H; thus, two kilograms will be converted to energy:

Energy = \( m c^2 = (2.00 \text{ kg})(2.99792 \times 10^8 \text{ m/s})(1 \text{ J/kg/m/s}^2) = 1.7975048 \times 10^{17} \text{ J} \) (unrounded)

\( \text{ H atoms} = \left( \frac{1.7975048 \times 10^{17}}{1 \text{ kg}} \right) \left( \frac{1 \text{ kg}}{10^3 \text{ g}} \right) \left( \frac{0.0078 \text{ g H}}{1 \text{ mol H}} \right) \left( \frac{1 \text{ mol H}}{6.022 \times 10^{23} \text{ atoms H}} \right) \left( \frac{2.0709066 \times 10^{-17}}{1 \text{ kg H}} \right) \)

\( = 1.4525903 \times 10^{17} = 1.45 \times 10^7 \text{ H atoms} \)

\( ^4_2 \text{He} + 2^0_1 \beta \) (Positrons have the same mass as electrons.)

\( \Delta m = [4(1.007825 \text{ amu})] - [4(0.00260 \text{ amu} + 2(0.000549 \text{ amu})]) = 0.027602 \text{ amu} \)

\( \Delta m = \left( \frac{0.027602 \text{ g}}{1 \text{ mol H}} \right) \left( \frac{1 \text{ kg}}{10^3 \text{ g}} \right) \left( \frac{1 \text{ mol H}}{4 \text{ mol H}} \right) \left( \frac{1 \text{ mol H}}{6.022 \times 10^{23} \text{ atoms H}} \right) \left( 1.4525903 \times 10^7 \text{ H atoms} \right) \)

\( = 1.6644967 \times 10^{-22} \text{ kg} \) (unrounded)

Energy = \( (\Delta m)c^2 = (1.6644967 \times 10^{-22} \text{ kg})(2.99792 \times 10^8 \text{ m/s})^2(1 \text{ J/kg/m/s}^2) = 1.4959705 \times 10^{-5} = 1.4960 \times 10^{-5} \text{ J} \)

d) Calculate the energy generated in part (b):

Energy = \( \left( \frac{1.7975048 \times 10^{17}}{1 \text{ kg H}} \right) \left( \frac{1 \text{ kg}}{10^3 \text{ g}} \right) \left( \frac{0.0078 \text{ g H}}{1 \text{ mol H}} \right) \left( \frac{1 \text{ mol H}}{6.022 \times 10^{23} \text{ atoms H}} \right) \left( 3.0081789 \times 10^{-10} \text{ J} \right) \)

\( = 3.0081789 \times 10^{-10} \text{ J} \)

Energy increase = \( (1.4959705 \times 10^{-5} - 3.0081789 \times 10^{-10} \text{ J}) = 1.4959404 \times 10^{-5} = 1.4959 \times 10^{-5} \text{ J} \)
e) $^3_1\text{H} \rightarrow ^3_2\text{He} + ^0_1\beta$

$$\Delta m = [3(1.007825 \text{ amu})] - [3.01603 \text{ amu} + 0.000549 \text{ amu}]$$

$$= 0.006896 \text{ g/mol}^3_2\text{He}$$

$$\Delta m = \left(\frac{0.006896 \text{ g}}{\text{mol He}}\right)\left(\frac{1 \text{ kg}}{10^3 \text{ g}}\right)\left(\frac{1 \text{ mol He}}{3 \text{ mol H}}\right)\left(\frac{1 \text{ mol H}}{6.022 \times 10^{23} \text{ H atoms}}\right)\left(1.4525903 \times 10^7 \text{ H atoms}\right)$$

$$= 5.5447042 \times 10^{-23} \text{ kg (unrounded)}$$

$$\text{Energy} = (\Delta m)c^2 = (5.5447042 \times 10^{-23} \text{ kg})(2.99792 \times 10^8 \text{ m/s})^2(J / (\text{kg} \cdot \text{m}^2/\text{s}^2))$$

$$= 4.9833164 \times 10^{-6} = 4.983 \times 10^{-6} \text{ J}$$

No, the Chief Engineer should advise the Captain to keep the current technology.

24.132 Multiply the equation $E = \frac{hc}{\lambda}$ by Avogadro’s number (N) to get the energy change per mole.

$$E = N\frac{hc}{\lambda} = \left(6.022 \times 10^{23} \text{ /mol}\right)\left(6.626 \times 10^{-34} \text{ J} \cdot \text{s}\right)\left(2.99792 \times 10^8 \text{ m/s}\right)\left(\frac{1 \text{ pm}}{10^{-12} \text{ m}}\right)$$

$$= 1.3702442 \times 10^{19} \text{ J/mol (unrounded)}$$

Determine the mass through a series of conversions from the chapter and the inside back cover:

$$\text{mass} = \left(\frac{1.3702442 \times 10^{19} \text{ J}}{1 \text{ MeV}}\right)\left(\frac{1 \text{ amu}}{931.5 \text{ MeV}}\right)\left(1.602 \times 10^{-19} \text{ J}/\text{amu}\right)$$

$$= 1.52476 \times 10^{-7} = 1.52 \times 10^{-7} \text{ kg/mol}$$

24.133 $k = (\ln 2) / t_{1/2} = (\ln 2) / (5.27 \text{ yr}) = 0.131526979 \text{ yr}^{-1} \text{ (unrounded)}$

$$\frac{\ln N_t}{N_0} = -kt$$

$$\ln \left(\frac{70\%}{100\%}\right) = -(0.131526979 \text{ yr}^{-1})t$$

$$-0.356674943 = -(0.131526979 \text{ yr}^{-1})t$$

$$t = 2.71180 \text{ years (unrounded)}$$

The source must be replaced after the time calculated above. Two years would be March 1, 2009. Then add 0.7 years which is $(365.25 \times 0.7) = 256 \text{ days}$ (actually the significant figures limit this to $3 \times 10^2 \text{ days}$). The final date is November 12, 2009.

24.134 The difference in the energies of the two $\alpha$ particles gives the energy of the $\gamma$-ray released to get from excited state I to the ground state. The energy of the 2% $\alpha$ particle is equal to the highest energy $\alpha$ particle minus the energy of the two $\gamma$-rays (the rays are from excited state II to excited state I, and from excited state I to the ground state).

a) $(4.816 - 4.773) \text{ MeV} = 0.043 \text{ MeV} = \text{energy of } \gamma\text{-ray}$. Use $E = \frac{hc}{\lambda}$ to determine the wavelength.

$$\lambda = \frac{\left(6.626 \times 10^{-34} \text{ J} \cdot \text{s}\right)\left(2.99792 \times 10^8 \text{ m/s}\right)}{0.043 \text{ MeV}} = 2.8836 \times 10^{-11} = 2.9 \times 10^{-11} \text{ m}$$

b) $(4.816 - 0.043 - 0.060) \text{ MeV} = 4.713 \text{ MeV}$

24.135 a) Student 2 is correct. The fraction of uranium-238 remaining is equal to the amount of uranium-238 remaining divided by the initial amount. The initial amount is equal to the sum of the amount of uranium-238 remaining plus the amount of lead-206 that resulted from the decay of uranium-238.

b) amount $U_{238} = 2 \text{ (Pb-206)}$

$$2x = x$$

$$\text{fraction } U_{238} = \frac{238\text{U}}{238\text{U} + \text{Pb-206}} = \frac{2x}{2x + x} = \frac{2x}{3x} = \frac{2}{3}$$

$$k = (\ln 2) / t_{1/2} = (\ln 2) / (4.5 \times 10^9 \text{ yr}) = 1.540327 \times 10^{-10} \text{ yr}^{-1} \text{ (unrounded)}$$
\[
\ln \left( \frac{N_t}{N_0} \right) = -kt
\]

\[
\ln \left( \frac{2/3}{1} \right) = -(1.540327 \times 10^{-10} \text{ yr}^{-1})t
\]

\[
-0.405465108 = -(1.540327 \times 10^{-10} \text{ yr}^{-1})t
\]

\[
t = 2.63233 \times 10^9 = 2.6 \times 10^9 \text{ years old}
\]

24.136 \( ^{232} \text{Th} \to ^{228} \text{Ra} + \frac{4}{3} \alpha \)
\( ^{228} \text{Ra} \to ^{228} \text{Ac} + \frac{2}{3} \beta \)
\( ^{228} \text{Ac} \to ^{228} \text{Th} + \frac{2}{3} \beta \)
\( ^{232} \text{Th} \to ^{232} \text{Ra} + \frac{4}{3} \alpha \)
\( ^{224} \text{Ra} \to ^{224} \text{Rn} + \frac{2}{3} \alpha \)
\( ^{220} \text{Rn} \to ^{216} \text{Po} + \frac{4}{3} \alpha \)
\( ^{216} \text{Po} \to ^{212} \text{Pb} + \frac{2}{3} \alpha \)
\( ^{212} \text{Pb} \to ^{212} \text{Bi} + \frac{2}{3} \beta \)
\( ^{212} \text{Bi} \to ^{212} \text{Po} + \frac{2}{3} \beta \)
\( ^{212} \text{Po} \to ^{208} \text{Pb} + \frac{2}{3} \alpha \)

24.137 The age of Egyptian mummies is on the order of a few thousand years. An isotope with a half-life of a few thousand years would be the best choice. Carbon-14, with a half life of 5730 years, would be the best choice.

24.138 Multiply each of the half lives by 20 (the number of half lives is considered to be exact).

a) \( ^{242} \text{Cm} \)
\( 20 \times (163 \text{ days}) = 3.26 \times 10^3 \text{ days} \)

b) \( ^{214} \text{Po} \)
\( 20 \times (1.6 \times 10^{-4} \text{ s}) = 3.2 \times 10^{-3} \text{ s} \)

c) \( ^{232} \text{Th} \)
\( 20 \times (1.39 \times 10^{10} \text{ yr}) = 2.78 \times 10^{11} \text{ yr} \)

24.139 If the blade was made in 100 AD it is about \( 1.9 \times 10^3 \) years old.

\( k = (\ln 2) / t_{1/2} = (\ln 2) / 5730 \text{ yr} = 1.2096809 \times 10^{-4} \text{ yr}^{-1} \) (unrounded)

\[
\ln \left( \frac{N_t}{N_0} \right) = -kt
\]

Handle:
\[
\ln \left( \frac{10.1 \text{ d/min} \times \text{ g}}{15.3 \text{ d/min} \times \text{ g}} \right) = -(1.2096809 \times 10^{-4} \text{ yr}^{-1})t
\]

\[
-0.415317404 = -(1.2096809 \times 10^{-4} \text{ yr}^{-1})t
\]

\[
t = 3.43328 \times 10^3 = 3.43 \times 10^3 \text{ yr}
\]

Inlaid:
\[
\ln \left( \frac{13.8 \text{ d/min} \times \text{ g}}{15.3 \text{ d/min} \times \text{ g}} \right) = -(1.2096809 \times 10^{-4} \text{ yr}^{-1})t
\]

\[
-0.103184236 = -(1.2096809 \times 10^{-4} \text{ yr}^{-1})t
\]

\[
t = 8.52987 \times 10^2 = 8.53 \times 10^2 \text{ yr}
\]

Ribbon:
\[
\ln \left( \frac{12.1 \text{ d/min} \times \text{ g}}{15.3 \text{ d/min} \times \text{ g}} \right) = -(1.2096809 \times 10^{-4} \text{ yr}^{-1})t
\]

\[
-0.103184236 = -(1.2096809 \times 10^{-4} \text{ yr}^{-1})t
\]

\[
t = 1.939746 \times 10^3 = 1.94 \times 10^3 \text{ yr}
\]
Sheath:

\[
\ln \left( \frac{15.0 \text{ d/min} \cdot \text{g}}{15.3 \text{ d/min} \cdot \text{g}} \right) = -(1.2096809 \times 10^{-4} \text{ yr}^{-1}) t
\]

\[
-0.103184236 = -(1.2096809 \times 10^{-4} \text{ yr}^{-1}) t
\]

\[
t = 1.63701 \times 10^2 = 1.64 \times 10^2 \text{ yr}
\]

The ribbon is nearest in age to the blade.

24.140 a) When 1.00 kg of antimatter annihilates 1.00 kg of matter, the change in mass is:

\[
\Delta m = 0 - 2.00 \text{ kg} = -2.00 \text{ kg}
\]

The energy released is calculated from \( \Delta E = \Delta mc^2 \).

\[
\Delta E = (-2.00 \text{ kg}) \left( 2.99792 \times 10^8 \text{ m/s} \right)^2 \left( \text{J/(kg}\cdot\text{m}^2/\text{s}^2) \right) = -1.7975048 \times 10^{17} = -1.80 \times 10^{17} \text{ J}
\]

The negative value indicates the energy is released.

b) Assuming that four hydrogen atoms fuse to form the two protons and two neutrons in one helium atom and release two positrons, the energy released can be calculated from the binding energy of helium-4.

\[
4^1\text{H} \rightarrow ^4\text{He} + 2^0\text{e}_\beta
\]

\[
\Delta m = [4 \times (1.007825 \text{ amu})] - [4 \times 0.00260 \text{ amu} + 2 \times (0.000549 \text{ amu})]
\]

\[
= 0.02760 \text{ amu per } ^4\text{He} \text{ formed}
\]

\[
\text{total } \Delta m = (1.00 \times 10^5 \text{ H atoms}) \left( \frac{0.02760 \text{ amu}}{^4\text{He}} \right) \left( \frac{1^4\text{He}}{4 \text{ H atoms}} \right) = 6.90 \times 10^2 \text{ amu}
\]

\[
\text{Energy} = \left( 6.90 \times 10^2 \text{ amu} \right) \left( \frac{931.5 \text{ MeV}}{1 \text{ amu}} \right) \left( \frac{1.602 \times 10^{-16} \text{ kJ}}{1 \text{ MeV}} \right)
\]

\[
= 1.02966 \times 10^{-10} \text{ kJ per anti-H collision (unrounded)}
\]

\[
\text{Anti-H atoms} = (1.00 \text{ kg}) \left( \frac{10^3 \text{ g}}{1 \text{ kg}} \right) \left( \frac{1 \text{ mol anti- } ^1\text{H}}{1.008 \text{ g anti- } ^1\text{H}} \right) \left( \frac{6.022 \times 10^{23} \text{ anti- } ^1\text{H}}{1 \text{ mol anti- } ^1\text{H}} \right)
\]

\[
= 5.9742 \times 10^{26} \text{ anti-H (unrounded)}
\]

\[
\text{Energy released} = \left( \frac{1.02966 \times 10^{-10} \text{ kJ}}{\text{anti- } ^1\text{H}} \right) \left( 5.9742 \times 10^{26} \text{ anti- } ^1\text{H} \right) = 6.15139 \times 10^{16} = 6.15 \times 10^{16} \text{ kJ}
\]

c) From the above calculations, the procedure in part b) with excess hydrogen produces more energy per kilogram of antihydrogen.

24.141 Einstein’s equation is \( E = mc^2 \), which is modified to \( E = \Delta mc^2 \) to reflect a mass defect. The speed of light, \( c \), is \( 2.99792 \times 10^8 \text{ m/s} \). The mass of exactly one amu is \( 1.66054 \times 10^{-27} \text{ kg} \) (inside back cover). When the quantities are multiplied together, the unit will be \( \text{kg}\cdot\text{m}^2/\text{s}^2 \), which is also the unit of joules. Convert J to MeV using the conversion factor \( 1.602 \times 10^{-13} \text{ J} = 1 \text{ MeV} \).

\[
\Delta E = (\Delta m)c^2
\]

\[
= \left( 1 \text{ amu} \right) \left( \frac{1.66054 \times 10^{-27} \text{ kg}}{1 \text{ amu}} \right) \left( 2.99792 \times 10^8 \text{ m/s} \right)^2 \left( \frac{1 \text{ MeV}}{1.602 \times 10^{-13} \text{ J}} \right)
\]

\[
= 9.3159448 \times 10^2 = 9.316 \times 10^2 \text{ MeV}
\]

24.142 The original amount of uranium is the current amount plus the amount converted to lead. We need to determine the amount of decayed uranium from the lead in the sample:

\[
\text{mass of } ^{238}\text{U} = \left( 0.023 \text{ g } ^{206}\text{Pb} \right) \left( \frac{1 \text{ mol } ^{206}\text{Pb}}{206 \text{ g } ^{206}\text{Pb}} \right) \left( \frac{1 \text{ mol } ^{238}\text{U}}{1 \text{ mol } ^{206}\text{Pb}} \right) \left( \frac{238 \text{ g } ^{238}\text{U}}{1 \text{ mol } ^{238}\text{U}} \right)
\]

\[
= 0.0265728 \text{ g } ^{238}\text{U} \text{ (unrounded)}
\]

Original mass of \( ^{238}\text{U} = (0.065 + 0.0265728) \text{ g } ^{238}\text{U} = 0.0915728 \text{ g } ^{238}\text{U} \text{ (unrounded)}
\]

\[
k = (\ln 2) / t_{1/2} = (\ln 2) / 4.5 \times 10^9 \text{ yr} = 1.540327 \times 10^{-10} \text{ yr}^{-1} \text{ (unrounded)}
\]

24-25
\[
\ln \left( \frac{N_1}{N_0} \right) = -kt
\]

\[
\ln \left( \frac{0.065 \text{ g}}{0.0915728 \text{ g}} \right) = -(1.540327 \times 10^{-10} \text{ yr}^{-1})t
\]

\[-0.342747014 = -(1.540327 \times 10^{-10} \text{ yr}^{-1})t
\]

\[t = \frac{0.342747014}{1.540327 \times 10^{-10} \text{ yr}^{-1}} = 2.2251574 \times 10^9 = 2.2 \times 10^9 \text{ years old}
\]

24.143 a) \(^{242}\text{Pu} \rightarrow ^{238}\text{U}^* + 4^2\alpha\)

\(^{238}\text{U}^* \rightarrow ^{238}\text{U} + \gamma\)

b) Determine the MeV of the \(\gamma\)-ray by using \(E = hc / \lambda\) to determine the energy.

\[
E = \frac{hc}{\lambda} = \left( \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{2.99792 \times 10^8 \text{ m/s}} \right) \left( \frac{1 \text{ nm}}{10^{-9} \text{ m}} \right) = \frac{1 \text{ MeV}}{1.602 \times 10^{-13} \text{ J}} = 0.044975 \text{ MeV (unrounded)}
\]

Adding the two energies together gives the energy required to go directly from plutonium-242 to the lowest energy uranium-238:

Total energy = (4.853 + 0.044975) MeV = 4.897975 = \textbf{4.898 MeV}

24.144 \(^{249}\text{Cf} + ^{18}\text{O} \rightarrow ^{263}\text{Sg} + 4^1\text{n}\)

\(^{263}\text{Sg} \rightarrow ^{259}\text{Rf} + 4^2\alpha\)

\(^{259}\text{Rf} \rightarrow ^{255}\text{No} + 4^2\alpha\)

\(^{255}\text{No} \rightarrow ^{251}\text{Fm} + 4^2\alpha\)

24.145 The rate of formation of plutonium-239 depends on the rate of decay of neptunium-239 with a half-life of 2.35 days.

\(k = (\ln 2) / t_{1/2} = (\ln 2) / 2.35 \text{ days} = 0.294956 \text{ day}^{-1} \text{ (unrounded)}\)

\[
\ln \left( \frac{N_1}{N_0} \right) = -kt
\]

\[
\ln \left( \frac{(1.00 \text{ kg}) (90\% / 100\%)}{1.00 \text{ kg}} \right) = -(0.294956 \text{ day}^{-1})t
\]

\[-0.105360515 = -(0.294956 \text{ day}^{-1})t
\]

\[t = \frac{0.105360515}{0.294956 \text{ day}^{-1}} = 0.357207568 = 0.357 \text{ days}
\]

24.146 a) Half of the atoms do not remain after each half-life. Decay is a random process. On average half of the atoms remain after 1 half-life, but individual simulations may vary.

b) Increasing the number of atoms is more realistic. The number of remaining atoms varies less. Also, a real radioactive sample consists of many atoms.

24.147 a) Ideally 128 atoms remain after the first half-life, 64 atoms remain after the second, 32 atoms remain after the third, 16 atoms remain after the fourth, and 8 remain after the fifth.

b) To make the simulation more realistic, make the size of the sample (# of atoms) much larger.

24.148 a) Nucleus 1 is \(^9\text{Be}\); Nucleus 2 is \(^{10}\text{Be}\); Nucleus 3 is \(^7\text{Be}\)

b) The n/p ratio for the stable Nucleus 1 = 5/4 = 1.25; the n/p ratio for Nucleus 2 = 6/4 = 1.5. Nucleus 2 has an n/p ratio that is too high and the most likely mode of decay is beta particle emission. The n/p ratio of Nucleus 3 = 3/4 = 0.75. This ratio is too low and the expected modes of decay are electron capture and/or positron emission.